Physics 1A Final Exam. June 2001. Closed Book.

Instructions:
1. Attempt all 6 questions; each question is worth 20 points, so answering 5 questions correctly will give you the maximum score of 100. Note that some questions may take longer than others to complete. Also, for most questions you may be able to answer parts (b) and (c) even if you cannot answer part (a).

2. Write in blue or black pen only; work in pencil will not be graded. Clearly "X" out any rough work (some students prefer to use one side of the blue book for rough work, and the facing page for graded work - this is a good idea).

3. Some advice: good diagrams will be given credit, as will correct explanations of which principle, law of motion etc. applies, even without calculation. If you cannot complete a calculation, try to write something *accurate* and *relevant* to the question in clear English sentences to receive partial credit.

4. You have 3 hours - good luck and DON'T PANIC!

General formulae:
Earth's gravity $g = 10\ \text{m/s}^2$; gravitational $P.E. = mgh$; $K.E. = \frac{1}{2} mv^2$
Friction force $F_F \leq \mu F_N$.
Angular speed $\omega = v/r = 2\pi f = 2\pi/T$.
Constant accel.: $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2} at^2$; $v^2 = v_0^2 + 2a(x - x_0)$

1. A hardened steel bullet of mass 0.01kg is fired horizontally into the vertical face of a 2kg block of wood, initially at rest. The bullet embeds itself into the block and both move forward with a common speed of 2.4m/s.

a. (i) State the difference between an elastic and an inelastic collision. (ii) Use momentum conservation to show that the bullet's initial speed was about 480 m/s; use 4 significant figures.

b. How much kinetic energy was lost in the collision?

c. Without calculation, but with the aid of a diagram, describe the subsequent motion of the block and bullet, if the bullet were composed of hard rubber (i.e. if the collision were elastic).

2. A 100kg rectangular container is unloaded from a ship onto the top of an 8m ramp, which is inclined at 30 degrees to the horizontal. The coefficient of friction is 0.5.

a. Show that the maximum frictional force between the box and the ramp is about 433 N; give 4 significant figures.

b. Therefore (i) how much force must a worker exert on the box to prevent it from sliding down the ramp? (ii) What minimum force would be required to push the box *up* the ramp?

c. The worker hears the lunch bell and releases the box. What is the resulting speed of the box at the bottom of the ramp? (Use either Newton's laws, or work-energy arguments).

Earth's gravity $g = 10\ \text{m/s}^2$; gravitational $P.E. = mgh$; $K.E. = \tfrac{1}{2}mv^2$
Friction force $F_F \le \mu F_N$.
Constant accel.: $v = v_0 + at$; $x = x_0 + v_0 t + \tfrac{1}{2}at^2$; $v^2 = v_0^2 + 2a(x - x_0)$

3. On the same ramp as in question 2 (8m long, 30 degree incline), another worker allows a solid cylindrical drum to roll down the ramp, starting from rest at the top. The drum has mass 100kg. radius 0.2m, and moment of inertia I=2.0 kgm^2.

a. When rolling on any surface without slipping, show that 1/3 of the drum's *total* kinetic energy is stored as rotatonal energy.

b. Therefore, using energy arguments, find the speed of the drum at the bottom of the ramp. (Neglect any rolling friction).

c. The drum rolls off the end of the dock and falls, still spinning, into the water. If the water exerts a total frictional force around the drum's surface of 180N, for how long does the drum continue to spin? (Hint: find the drum's angular momentum, then set equal to the {torquextime} required to change it to zero).

Earth's gravity $g = 10\ \text{m/s}^2$; gravitational $P.E. = mgh.$
Translational $K.E. = \tfrac{1}{2}mv^2$. Rotational $K.E. = \tfrac{1}{2}I\omega^2$.
Angular speed $\omega = v/r$. Angular momentum $L = I\omega$. Torque $\tau = rF\sin\theta = \frac{dL}{dt}$.

4. A student attaches a 0.2kg ball bearing to a light vertical spring, which extends its length by 0.025m. She then sets the system into simple harmonic motion with an initial speed of v_0 =0.32 m/s.

a. Show that (i) the spring constant is 80 N/m, and (ii) the vibration period is 0.314s.

b. Using energy arguments or otherwise, what is the resulting amplitude of the oscillation?

c. What is (i) the downward acceleration of the ball at the top of it's motion? Therefore, (ii) what is the effective weight of a flea (of negligible mass m) resting on top of the ball bearing at that moment? (i.e. Calculate the normal force of the ball bearing pushing up on the flea at this instant).

For a spring: $F = -kx$, $P.E. = \frac{1}{2}kx^2$, $\omega^2 = k/m$
Angular speed $\omega = 2\pi f = 2\pi/T$.

5. A car race is held on a horizontal circular track of inner radius 150m. The cars' tires have a coefficient of (sideways) sliding friction of 0.4.

a. What is (i) the maximum speed a car can drive around the inside track without skidding? Therefore, (ii) what is the corresponding minimum lap time?

In the same arena, a new race track is constructed which is banked at 25 degrees to the horizontal, such that the cars can now drive at similar speeds as before, even on wet or oil-covered portions of the track.

b. For a car driving around the 150m inner radius of the track, what speed should a driver attempt to maintain in slippery conditions, without relying on friction to keep the car from skidding?

c. What is the required power output of a car's engine (in kW) to maintain a speed of 30 m/s when the rolling friction+air resistance opposing the motion is 450N?

Friction force $F_F \leq \mu F_N$. Centripetal force $F_C = mv^2/r$. Circumference $= 2\pi r$

6. A sunken drum of mass 100kg is to be raised out of the water by a dockside crane.

a. (i) State Archimedes principle. (ii) The crane lifts the drum a few inches off the bottom and then stops. If the container's volume is 0.075m^3, show that the tension of the crane's cable at this moment is 250N.

b. The crane motor increases the tension to 300N, with the drum still underwater. What is the resulting instantaneous acceleration of the drum? (Neglect friction).

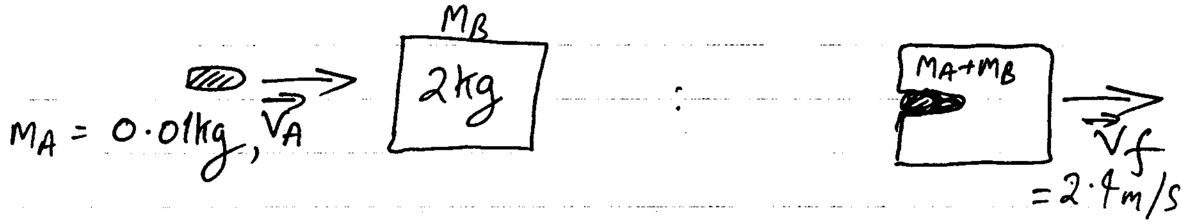
The crane lifts the drum 3m out of the water, and gently swings it sideways towards the dock. (This could be a pendulum problem, but it is not). However, the cable breaks when the drum is moving horizontally at 2.0 m/s at this height, and the drum falls freely back into the water.

c. (i) Show that the drum hits the water roughly 0.77s after release (give 3 decimal places). Therefore (ii) calculate the speed and angle at which it hits the water.

Buoyant force $F_B = \rho V g$. Density of water $\rho = 1000\text{kg/m}^3$.
Constant accel.: $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2} a t^2$; $v^2 = v_0^2 + 2a(x - x_0)$

Physics 1A Final Solutions

1.



a) i) In all collisions, momentum is conserved, but Kinetic energy is conserved only in elastic collisions

ii) Using conservation of momentum for this inelastic collision:

$$M_A V_A + M_B \overbrace{V_B}^{=0} = (M_A + M_B) V_f$$

i.e. $0.01 \times V_A = (0.01 + 2.0) \times 2.4$

$\Rightarrow V_A = 4.824 / 0.01 = \underline{482.4 \ m/s}$

b) Initial $KE = \frac{1}{2} M_A V_A^2 = \frac{1}{2} \times 0.1 \times 482.4^2 = 1163.55 J$

Final $KE = \frac{1}{2}(M_A + M_B) V_f^2 = \frac{1}{2} \times (0.01 + 2.0) \times 2.4^2 = 5.79 J$

So KE lost $= \underline{1157.76 \ J}$

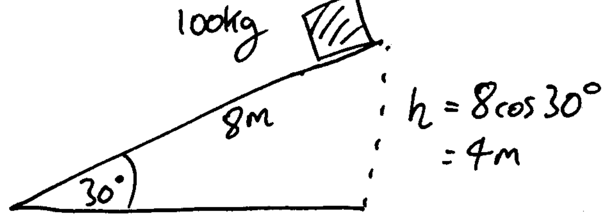
c) In the elastic case, K.E. and momentum conserved

After collision:

Rubber bullet rebounds off block with speed V_{Af}, slightly less than V_A. Total impulse $= M_A (V_{Af} - V_A)$

$\approx 2 M_A V_A$

Optional [Same impulse imparted to wooden block, knocking it forwards
In fact, impulse $= M_B V_{Bf} \approx 2 M_A V_A$
$\Rightarrow$ $V_{Bf} \approx 2 (M_A / M_B) V_A \approx 4.8 m/s$, $\approx 2 V_f$



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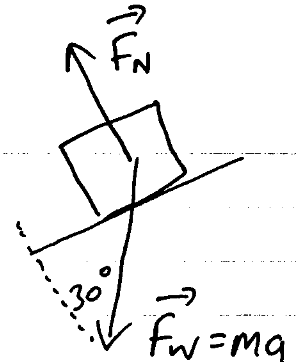
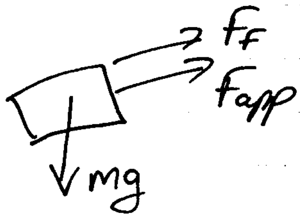


a) Force F_N normal to surface is balanced by component of $\vec{F_W}$.
i.e. $F_N = mg\cos\theta$. So max. friction force $F_F \leq \mu F_N$

$$\Rightarrow F_F \leq \mu mg\cos\theta = 0.5 \times 100 \times 10 \times \cos 30^\circ = \underline{433.01\,N}.$$

b). Equate forces $\parallel$ to surface:

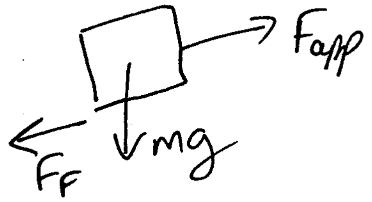


i) To keep box from sliding down

$$F_F + F_{app} = mg\sin\theta = 100 \times 10 \times 0.5 = 500\,N \text{ with } F_{app} \leq 433.01\,N$$

i.e. applied force $F_{app} = mg\sin\theta - \mu mg\cos\theta = 500 - 433 = \underline{67\,N}$

ii) To push box upwards, with friction opposing motion:



Now $F_{app} = mg\sin\theta + F_F$

$= mg\sin\theta + \mu mg\cos\theta = \underline{933\,N}$

c) PE lost (mgh) = KE gained $(\frac{1}{2}mv^2)$ + work against friction $(F_F . l$

i.e. $mgh = \frac{1}{2}mv^2 + \mu mgl\cos\theta$ with $h = l\sin\theta$

$\Rightarrow \quad v^2 = 2gh - 2\mu gl\cos\theta = 2gl(\sin\theta - \mu\cos\theta)$

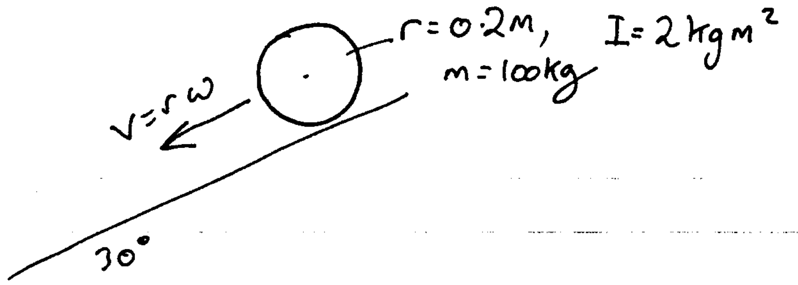
$\Rightarrow \underline{v = 3.277\,m/s}$

<u>OR</u> with $F_{app} = 0$, net accel. downwards $a = F/m = (g\sin\theta - \mu g\cos\theta$

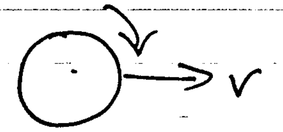
Then for constant accel. $v^2 = v_0^2 + 2al$

$\Rightarrow \quad v^2 = 0 + 2gl(\sin\theta - \mu\cos\theta)$ as above.

3.



a) On any surface, when rolling



$v = r\omega$

(Check: In 1 revolution, distance $= 2\pi r$, time $T = \frac{2\pi}{\omega}$

$\Rightarrow$ speed $v = 2\pi r . \omega / 2\pi = r\omega$)

$$\text{Total KE} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}v^2\left(m + I/r^2\right)$$

$$\text{So rotational / total K.E.} = \frac{I/r^2}{(m + I/r^2)} = \frac{\frac{2.0}{0.2^2}}{100 + 2.0/0.2^2} = \frac{50}{100+50}$$

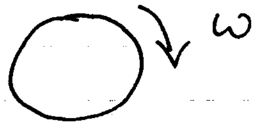
$= \underline{1/3}$, as required.

b) P.E. lost = total KE gained : $mgh = \tfrac{1}{2}v^2\left(m + I/r^2\right)$

with $h = l\sin 30° = 4m \Rightarrow v^2 = \dfrac{2gl\sin\theta}{\left(1 + I/mr^2\right)} = \dfrac{2\times10\times8\times0.5}{1+0.5}$

$\Rightarrow v = \sqrt{53.33} = \underline{7.30 \text{ m/s}}$.

c) Angular speed $\omega = v/r = \frac{7.30}{0.2} = 36.51$ rad/s



$\Rightarrow$ Ang. momentum $I\omega = 73.02 \text{ kgm}^2/\text{s}$

Force of 180N acts at distance r from rotation center

$\Rightarrow$ total torque $\tau = 180 \times 0.2 = 36$ Nm

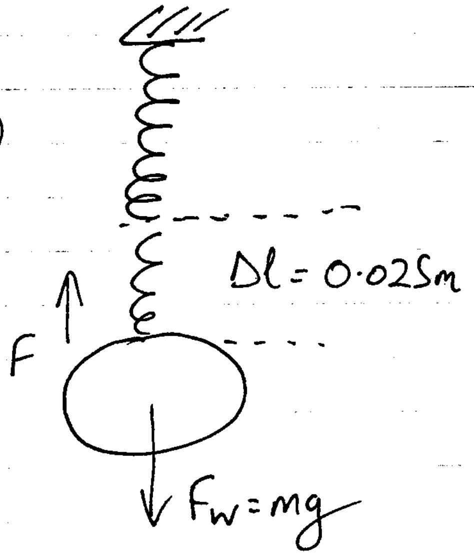
$\therefore$ to reduce ang. mom. to zero in time t

$$\tau t = \Delta L = I\omega \Rightarrow t = \frac{I\omega}{\tau} = \frac{73.02}{36} = \underline{2.02s}$$

Not required [(Ang. deceleration $\alpha = \tau / I = 18 \text{ rad/s}^2$, so drum turns through $\theta = \tfrac{1}{2}\alpha t^2 = 36.72$ rad or $(36.72/2\pi) = 5.84$ rotations)

4.

a)



(i) Hooke's law at equilibrium :

$\Rightarrow F_W = mg = k\Delta l$

So $k = \dfrac{mg}{\Delta l} = \dfrac{0.2 \times 10}{0.025} = \underline{80\ N/m}$

ii) When ball is displaced by x, restoring force $F = -kx = m\dfrac{d^2x}{dt^2}$

$\Rightarrow$ SHM with $\omega^2 = k/m$ and period $T = 2\pi/\omega$

$\therefore T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.2}{80}} = \pi/10 = \underline{0.314s}$

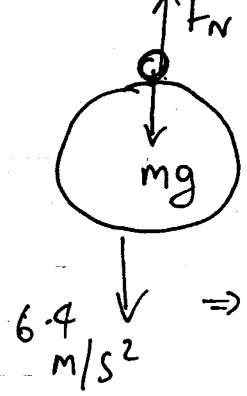
b) Initially, $KE = \frac{1}{2}mv_0^2$, $PE = 0$

When $x = A$, $PE = \frac{1}{2}kA^2$, $KE = 0$ (since $v=0$)

So total energy $\frac{1}{2}mv_0^2 = \frac{1}{2}kA^2 \Rightarrow A = v_0\sqrt{\dfrac{m}{k}}$

i.e. $A = 0.05v_0$ so with $v_0 = 0.32m/s$, $A = 0.016m$ or $\underline{1.6cm}$

c) At top of motion, downward accel. $\dfrac{d^2x}{dt^2} = (-)\dfrac{k}{m}A = 6.4m/s^2$



So for flea of mass m :

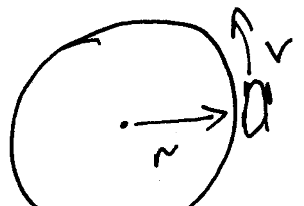
$ma = mg - F_N$

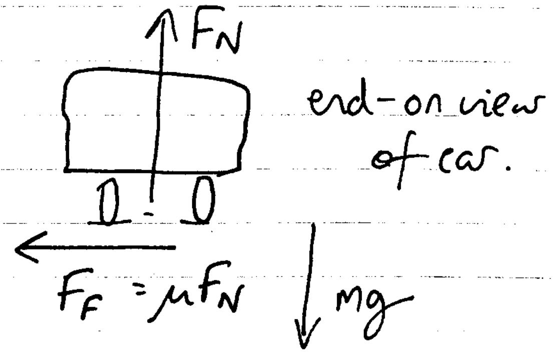
$\Rightarrow$ effective weight $F_N = m(g-a) = m(10-6.4)$

i.e. $F_N = 3.6m$ or 36% of static weight.

5. Top view of track:

a)





For car to stay moving in a circle

Friction force F_F = Centripetal force F_c

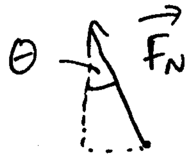
with $F_F \leq \mu F_N = \mu mg$, i.e. $\frac{mv^2}{r} \leq \mu mg$

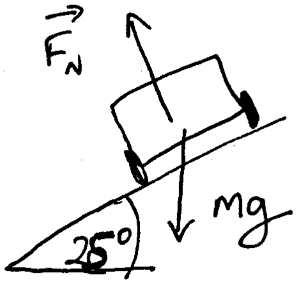
i) $\Rightarrow$ max speed $v^2 \leq \mu g r$ or $v \leq \sqrt{\mu g r}$

i.e. $v \leq \sqrt{0.4 \times 10 \times 150} = \sqrt{600} = \underline{24.5\ m/s}$

ii) So lap time $T = \frac{2\pi r}{v} \geq \frac{2\pi r}{\sqrt{\mu g r}} = 2\pi\sqrt{\frac{r}{\mu g}} = \underline{38.47s}$

b) End-on view.





Equate vertical forces

$\Rightarrow mg = F_N \cos\theta$

Equate horizontal forces, no friction $\Rightarrow F_c = \frac{mv^2}{r} = F_N \sin\theta$

$\Rightarrow \frac{mv^2}{r} = \frac{mg}{\cos\theta} . \sin\theta = mg \tan\theta$ with $\theta = 25°$

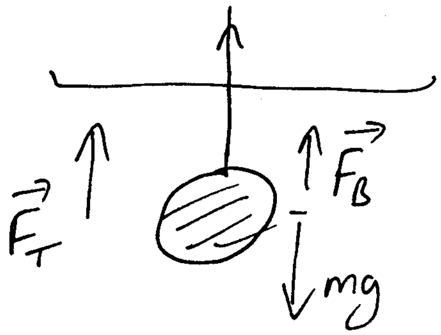
So speed now given by $v = \sqrt{rg\tan\theta} = \underline{26.44 m/s}$
(and lap time $T = \frac{2\pi r}{v} = 35.63\ s$)

c) Power = force x speed = $450N \times 30m/s = \underline{13.5\ kW}$.

6.

a i) "An object immersed in fluid experiences an upward buoyant force equal to the weight of water displaced"

ii)



$F_B = \rho V g$ so at equilibrium

$F_T + F_B = mg$

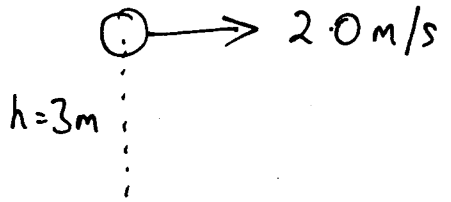
$\Rightarrow F_T = mg - \rho V g = g(m - \rho V) = 10 \times (100 - 1000 \times 0.075)$

$\Rightarrow F_T = 250\,N$

b) Net force upwards $F = F_T - (mg - \rho V g) = 300 - 250 = 50\,N$

$= \text{mass} \times \text{accel.}$ so $a = F/m = \frac{50}{100} = 0.5\,m/s$

c)

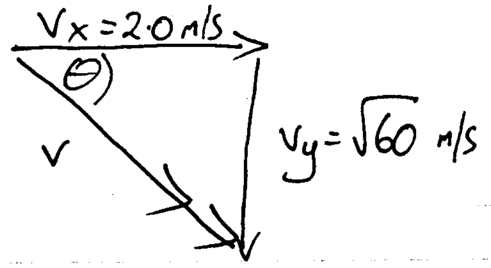


In free fall, time to hit water given by $h = \frac{1}{2} g t^2$

$\Rightarrow t = \sqrt{\frac{2h}{g}} = 0.7746\,s$

(ii) Vertical speed $V_y = gt$ or $V_y = \sqrt{2gh} = \sqrt{60} = 7.74\,m/$

So total speed



$v = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{60^2 + 2^2}$

$\Rightarrow v = 8\,m/s$, and angle $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) \Rightarrow \theta = 14.48°$