Test for Inertial Frames (week 2 lab)

Imagine a windowless lab

If lab moves with constant $\vec{v}$ i.e. $\frac{d\vec{v}}{dt} = 0$

(no acceleration or rotation)

then we can not tell whether we are moving or stationary

However, if "lab" accelerates $\frac{d\vec{v}}{dt} \neq 0$ $\frac{d\vec{v}}{dt}$

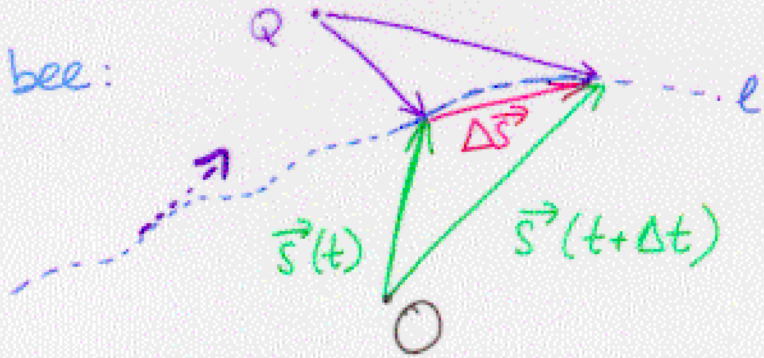
- can "feel" acceleration
- objects in lab starting with $\vec{v}$ = constant appear to now have $\frac{d\vec{v}}{dt} \neq 0$ (e.g. follow a curved path)

e.g. train car at uniform $\vec{v}$: throw keys up, fall straight down

" " accelerates : " " ,
keys "miss" hand on the way down

Vector Displacement and Velocity

e.g. path of a bee:



Displacement $\vec{s}$ (measured from O) changes with time

In interval Δt, $\vec{s} \rightarrow \vec{s} + \Delta\vec{s} = \vec{s}(t+\Delta t)$

<u>or</u> $\Delta\vec{s} = \vec{s}(t+\Delta t) - \vec{s}(t)$

Note: observer at Q measures different $\vec{s}(t)$ but <u>same $\Delta\vec{s}$</u>

Divide by time interval Δt $\rightarrow$ Velocity vector

$\vec{v} = \dfrac{\Delta\vec{s}}{\Delta t} =$ rate of change of displacement vector $\rightarrow \dfrac{d\vec{s}}{dt}$ as $\Delta t \rightarrow 0$.

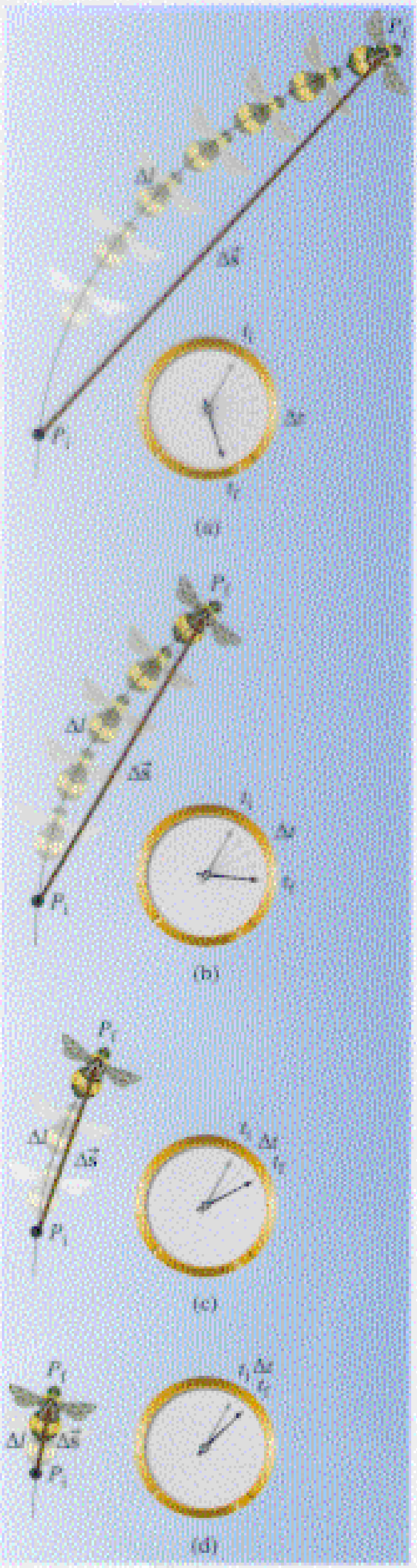
Properties :

- $\Delta\vec{s}$ is tangential to path $\Rightarrow$ $\vec{v}$ also tangent to path
- Different observers (O, Q) measure different $\vec{s}$ but same $\vec{v}$
- As $\Delta t \rightarrow 0$ magnitude $|\Delta\vec{s}| \rightarrow \Delta\ell$, path length

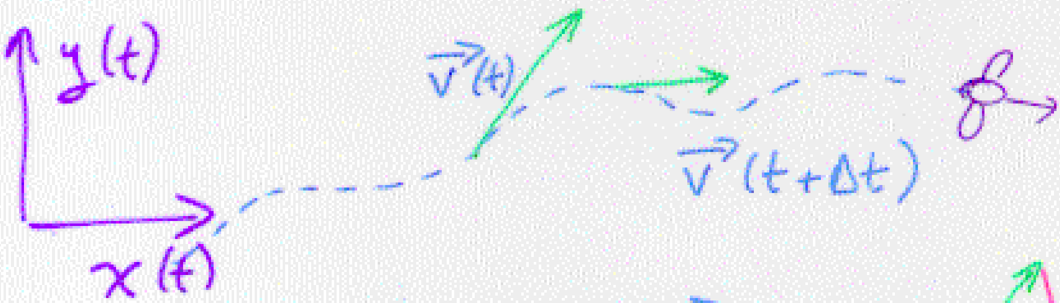
$\Rightarrow |\vec{v}| = \left|\dfrac{d\vec{s}}{dt}\right| \rightarrow \dfrac{d\ell}{dt}$, the speed.

6|2

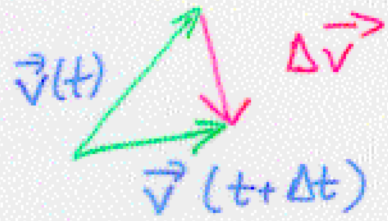
Figure 2.25
Instantaneous velocity of a bee



Acceleration = Rate of Change of Velocity



Change in velocity $\Delta \vec{v}$:



Can divide by Δt to get acceleration vector

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \frac{d\vec{v}}{dt} \text{ as } \Delta t \to 0$$

Notes:

- Components: If we write $\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} = V_x\vec{i} + V_y\vec{j}$

  Then $\vec{a} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j}$ and $|a| = \sqrt{a_x^2 + a_y^2}$ (m/s^2)

- Direction: $\vec{v}$ is tangential to path

  but $\vec{a} = \frac{d\vec{v}}{dt}$ can be any direction, even ⊥ to path of object

Acceleration due to Gravity: Free-Fall

Galileo: All objects falling under gravity have $|\vec{a}| = g$

3. A rock is thrown downwards from a tower. Shortly after release, neglecting air friction, it's downward acceleration:

a) is greater than g.

b) is equal to g.

c) is less than g.

d) depends on how fast it is thrown.

Initial height $= y_0$

" speed $= V_0 = \frac{dy}{dt} < 0$

Since accel. = constant, $\frac{d^2y}{dt^2} = -g$

$\Rightarrow V_y = V_0 - gt \quad \Rightarrow y = y_0 + V_0 t - \frac{1}{2} g t^2$

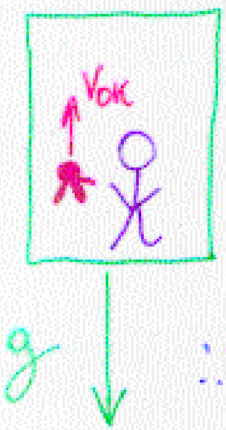
4. You are in an elevator when the support cable breaks and the elevator begins to free-fall. In a fright, you let go of your keys giving them a small upwards push. On the way down your keys will:

a) eventually drop to the floor.

b) eventually float up to the ceiling.

c) float up a small distance (eye level) and remain there.

d) accelerate and slam into the ceiling.



Relative to observer on ground

Keys: accel $= \frac{d^2y_K}{dt^2} = -g$, $V_{OK} > 0$

Passenger $= -g$, $V_{OP} = 0$

$\therefore$ for keys, $V_K = V_{OK} - gt$

passenger $V_P = 0 - gt$

Relative Speed $V_{KP} = V_K - V_P = V_{OK}$ (constant)

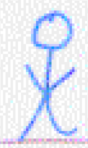
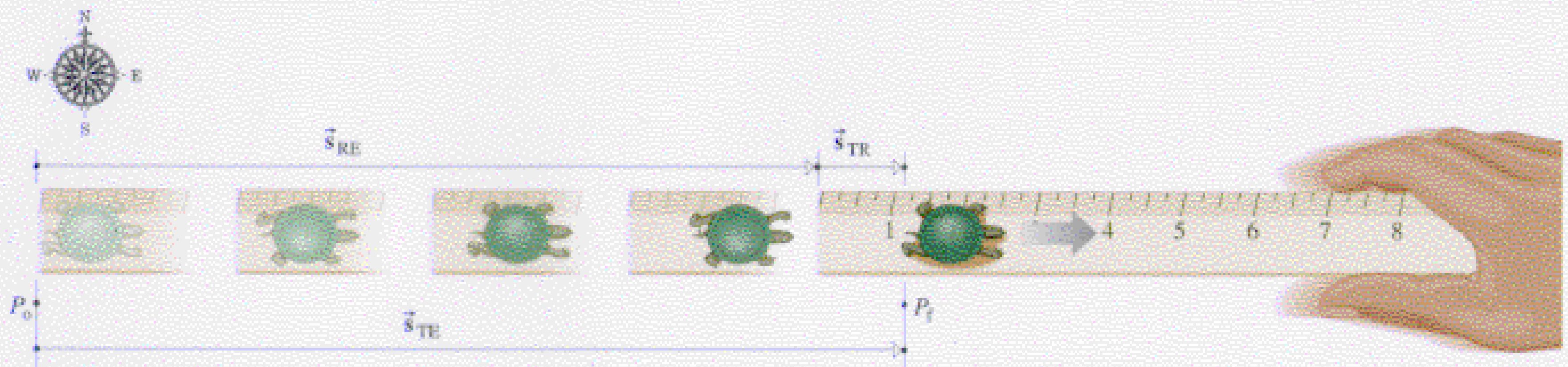


Figure 2.43

Displacement of turtle walking on a moving ruler



Turtle moves along ruler $\vec{S}_{TR} = \vec{S}_{TE} - \vec{S}_{RE}$ in time t

- does not depend on displacement of ruler (or earth)

∴ Relative position $\vec{S}_{TR}$ independent of absolute position

÷ by Δt:

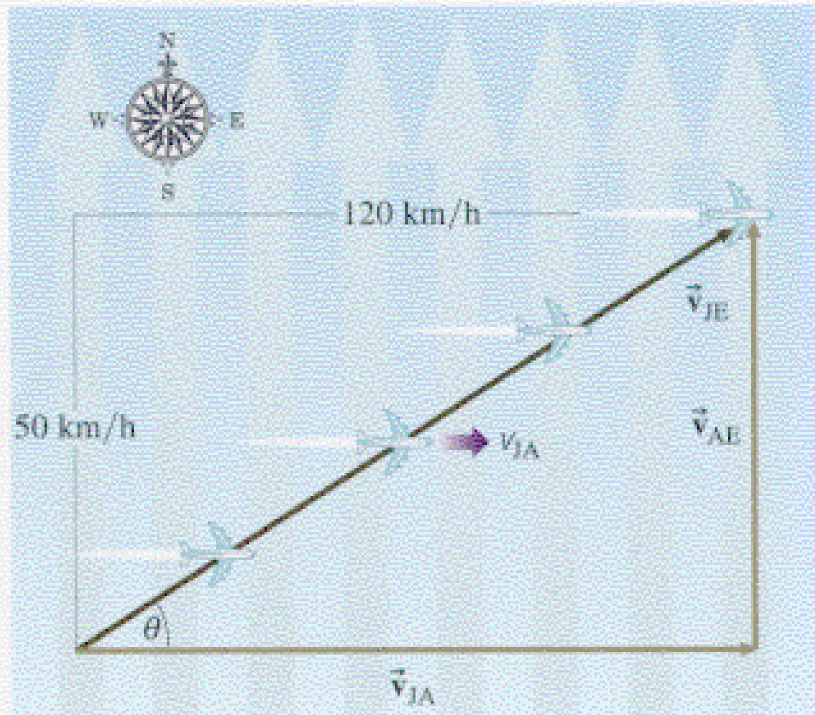
⇒ $\vec{V}_{TR} = \vec{V}_{TE} - \vec{V}_{RE}$

All inertial reference frames agree on $\vec{V}_{TR}$, and $\vec{V}_{TF} = \vec{V}_{TR} + \vec{V}_{RF}$

Figure 2.46

Plane heading east across a wind blowing north

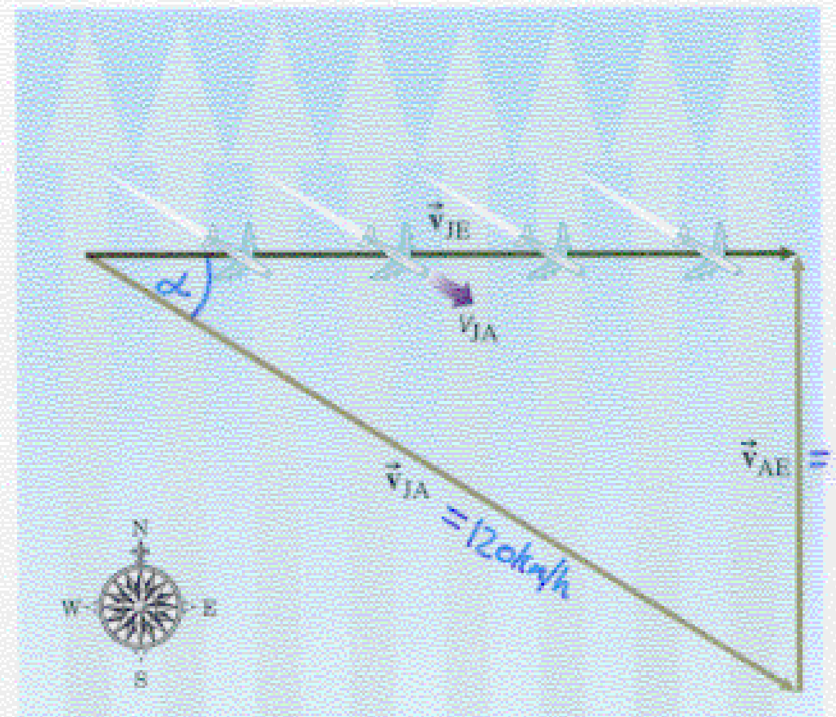
$$\vec{V}_{JE} = \vec{V}_{JA} + \vec{V}_{AE}$$



(a)

Ground speed $|\vec{V}_{JE}| = \sqrt{50^2 + 120^2} = 130\,km/h$

track $\theta = \tan^{-1}\left(\frac{50}{120}\right) = 22.6^\circ$ N of E



(b) - nose into wind

Ground speed $|\vec{V}_{JE}| = \sqrt{120^2 - 50^2} = 109.1\,km/h$

Heading required = $\sin^{-1}\left(\frac{50}{120}\right) = 24.6^\circ$ S of E