1-D Acceleration (Hecht ch.3, lab 1)

Note: Hecht 2.5 – 2.9 + all of Chapter 3 } Reading Quiz on Monday

Questions

Q: For our diver, how does (i) time in air and (ii) speed of impact depend on (a) initial height, (b) initial jump speed, and (c) "strength" of earth's gravity ?

Q: If speed is the gradient (i.e. rate of change) of position

$$v = \frac{dx}{dt}$$

– what should we call the rate of change of speed

$$a \equiv \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad ?$$

Scantron 20788

Acceleration $\equiv$ slope of $v(t)$

If $v(t)$ is a straight line:

$$a = \frac{v_1 - v_2}{t_1 - t_2}$$

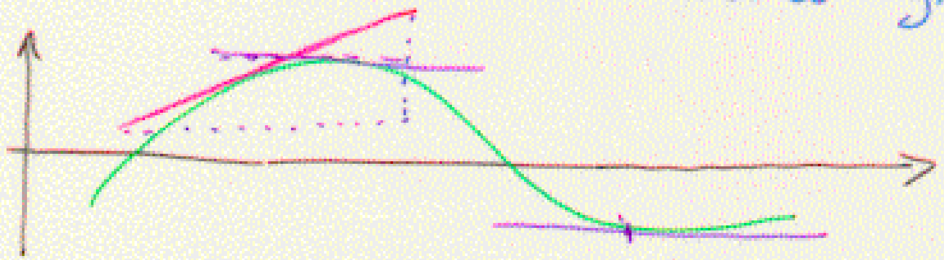
"deceleration"

$a < 0$ $\qquad$ $a > 0$

.... and if v is constant, accel. $a = 0$ ("coasting")

For general $v(t)$:

accel a = gradient of tangent



i.e. instantaneous accel. $a = \frac{dv}{dt}$

$\therefore$ When v is a max. or a min., $a = \frac{dv}{dt} = 0$

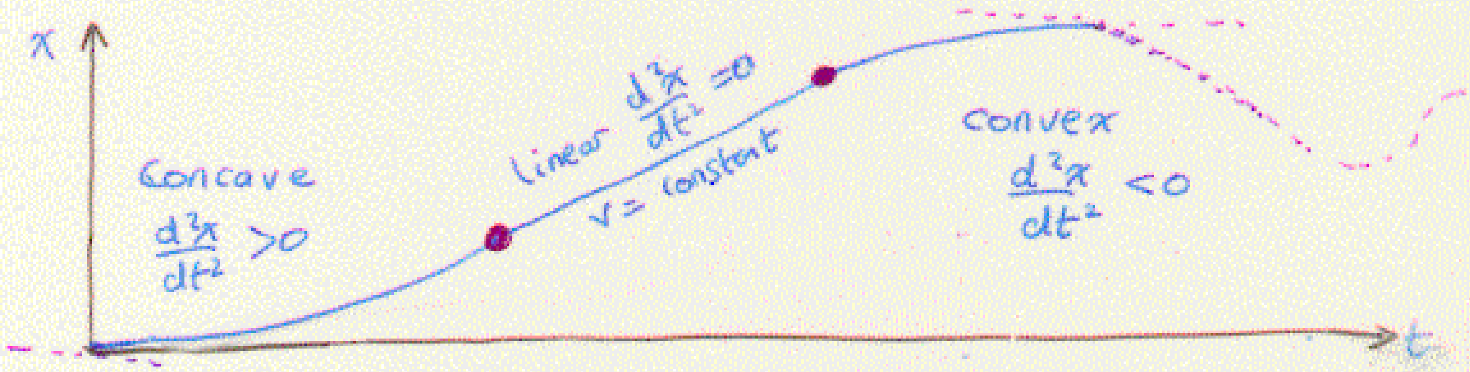
Note: Since $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

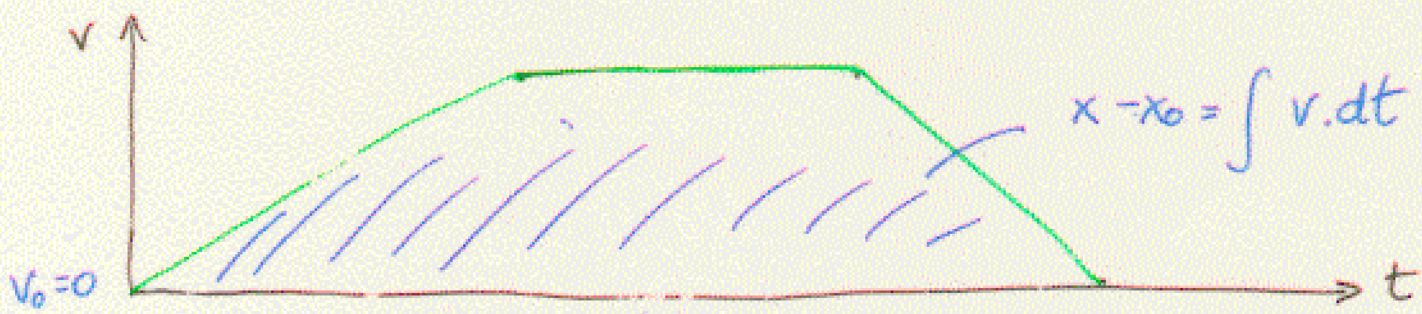
= "curvature" (or change of slope)

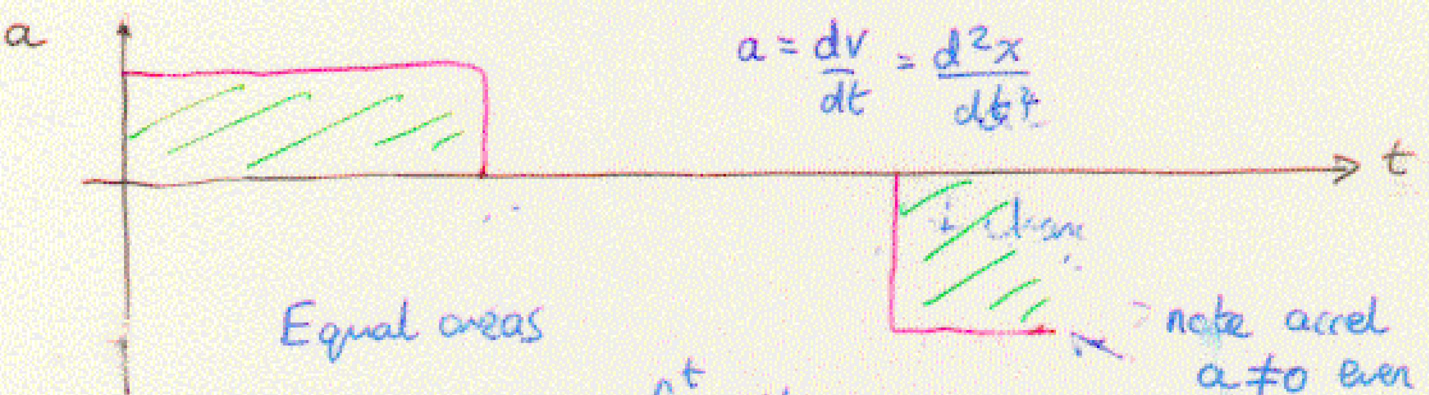
on graph of $x(t)$

Position, Speed, Acceleration

Eg. Astronaut "space walk" between Shuttle and Mir:



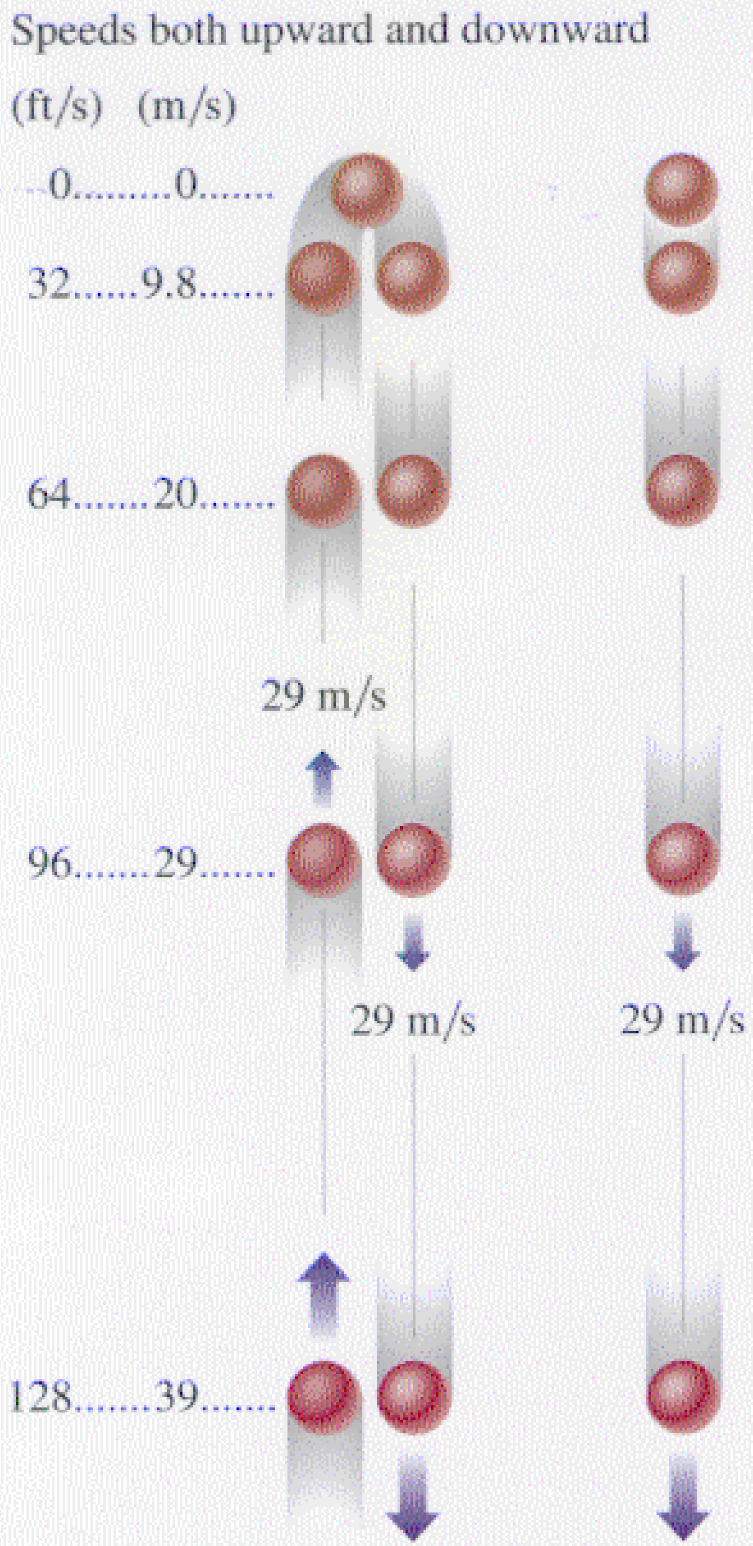




Equal areas

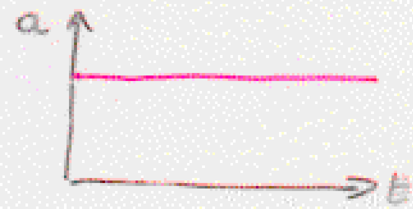
since $v - v_0 = \int_0^t a.dt = 0$

Figure 3.11
Peak altitude of a moving ball



Constant Acceleration: Equations of Motion

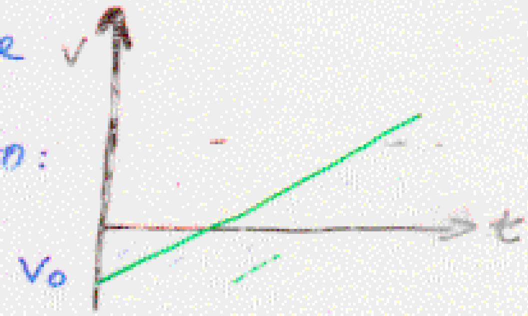
If $a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$ is constant



Then $v = \int a \cdot dt = v_0 + \int_0^t a \cdot dt = \underline{v_0 + at}$

i.e. $v(t)$ is a straight line

slope $= a$, intercept $= v_0$ at $t=0$:



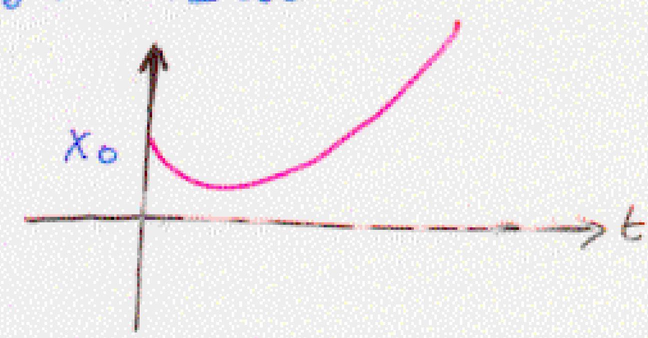
Then Position $x = \int v \cdot dt = \int \underline{(v_0 + at)} \cdot dt$

$$\Rightarrow \quad x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

i.e. $x(t)$ is a parabola

$x = x_0$ at $t = 0$

$v = v_0 = \frac{dx}{dt}$ at $t = 0$



Note: If $a > 0$, parabola concave (upwards)

$a < 0$ convex (downwards)

Constant Acceleration : Equations of Motion cont/d.

We have:

$$v = v_0 + at \qquad (1)$$

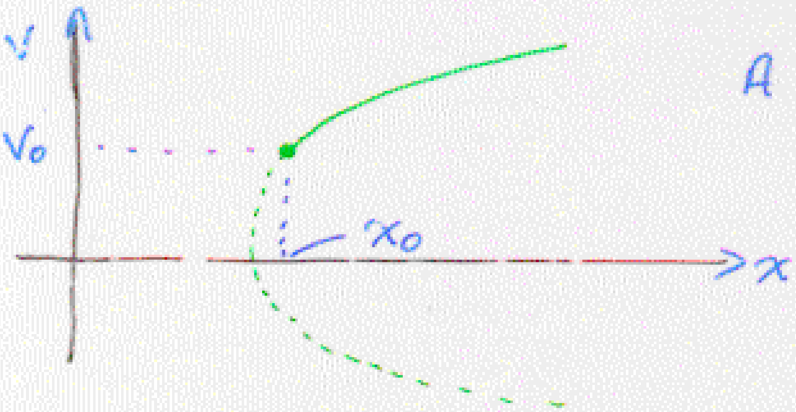
$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad (2)$$

Can we find speed v as a function of position too?

Yes! Eliminate 3rd variable "t" in ① and ②:

From ① : $t = \dfrac{v - v_0}{a}$: substitute into (2)

$$\Rightarrow \quad v^2 = v_0^2 + 2a(x - x_0) \qquad (3)$$



A (sideways) parabola

- Slope $\dfrac{dv}{dx} = \dfrac{\left(\frac{dv}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \dfrac{a}{v}$ at any point

- Be careful with sign of $v = \pm\sqrt{v_0^2 + 2a(x - x_0)}$

Applications: Free-fall (Physics at last!)

Galileo: All bodies fall at the same rate

(neglecting air friction)

Experiment:

As angle of plane ↑

acceleration a ↑, but

a = constant for given angle

Measure v, t, x to find a.

On earth, all bodies fall with constant acceleration

$$a = -g = -9.81\ \underline{\underline{m/s^2}} \qquad \frac{d^2x}{dt^2}$$

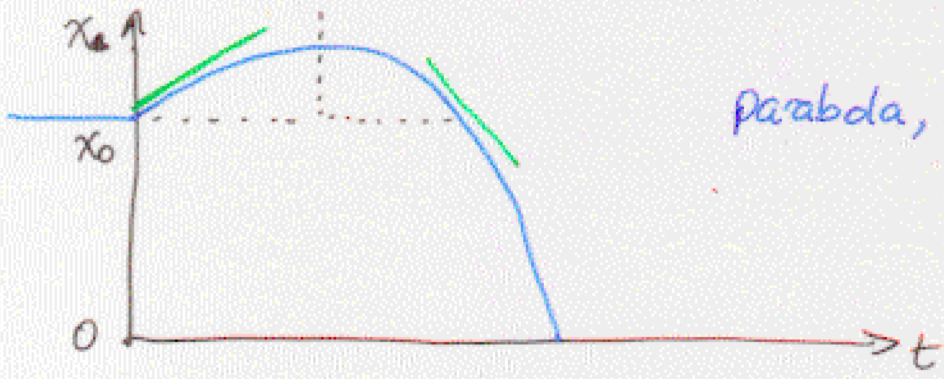
E.g. Drop object from initial height $x_0 = 5m$ ($v_0 = 0$)

$\Rightarrow$ height $\underline{x(t) = 5 - \frac{1}{2}gt^2}$ $= x_0 + v_0 t + \frac{1}{2} a t^2$

hits ground at $x = 0 \Rightarrow$ time $t = \sqrt{\dfrac{5}{\frac{1}{2}g}} = 0.504s$

$$\left.\begin{aligned} \text{at speed } v &= v_0 - gt \\ &= \sqrt{v_0^2 - 2g(x - x_0)} \end{aligned}\right\} = -4.95 m/s$$

Free-fall motion of diver under constant "g":



parabola, symmetric about peak x

$x_0 =$

$v_0 =$

Diver's height is a max. when $\frac{dx}{dt} = 0 = v = v_0 - gt$

i.e. $t\,(max) = v_0/g$

Max. height given by $v^2 = v_0^2 - 2g(x - x_0)_{max} = 0$

i.e. $x\,(max) = x_0 + \frac{v_0^2}{2g}$

When diver hits pool ($x = 0$):

Impact speed $v_i^2 = v_0^2 - 2g(-x_0) = \underline{v_0^2 + 2g x_0}$

Time of flight $t_i = \frac{(v_i - v_0)}{a} = \frac{(v_i - v_0)}{-g}$

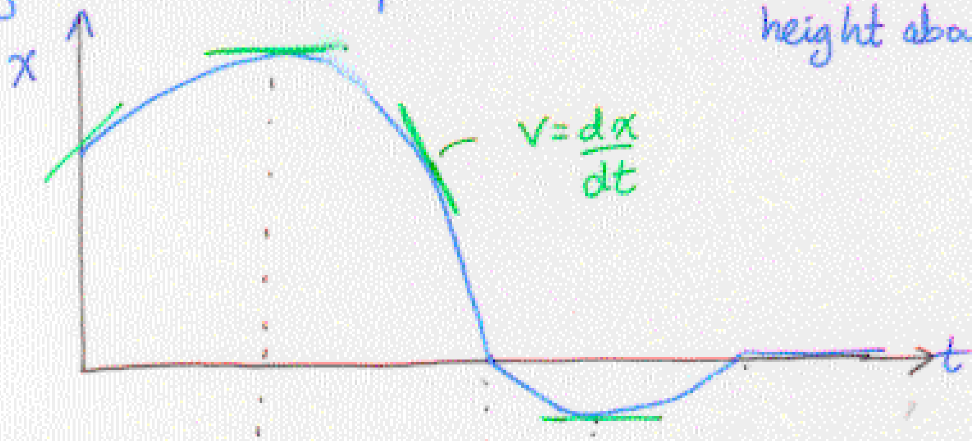
or given by solution of quadratic eqn:

$x = x_0 + v_0 t - \frac{1}{2} g t_i^2 = 0$

Optional: Read, don't copy! (See WWW notes)

e.g. Dive into pool:

height above pool ≡ position x



When we take the gradient to find $v = \frac{dx}{dt}$: