Questions to Ponder (DO NOT WRITE!)

- A man in a field tries to outrun a charging bull
  - he has a head start, but will he make it to the gate?

- A diver jumps off a springboard into the pool — when are the best moments to take a sharp picture of her?

- You are crossing a featureless Antarctic ice scape in a fancy 4WD expedition vehicle. The odometer freezes — can you tell how far you travelled that day?

- A train goes by and you observe a passenger walking along corridor inside. How fast is the passenger moving relative to you?

- (Dude, where's my TA???)

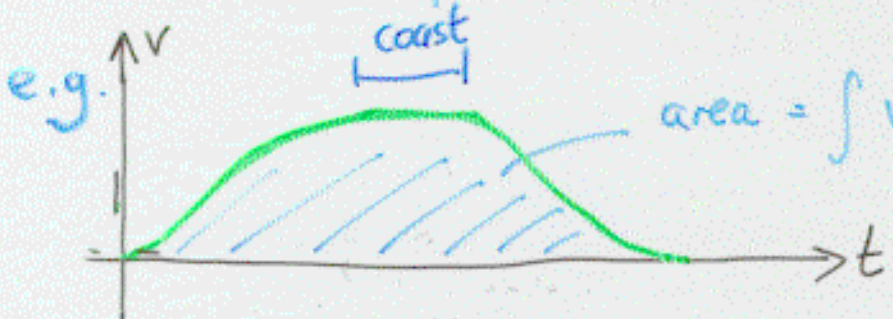
Distance (1-d displacement) from Speed – Integrate!

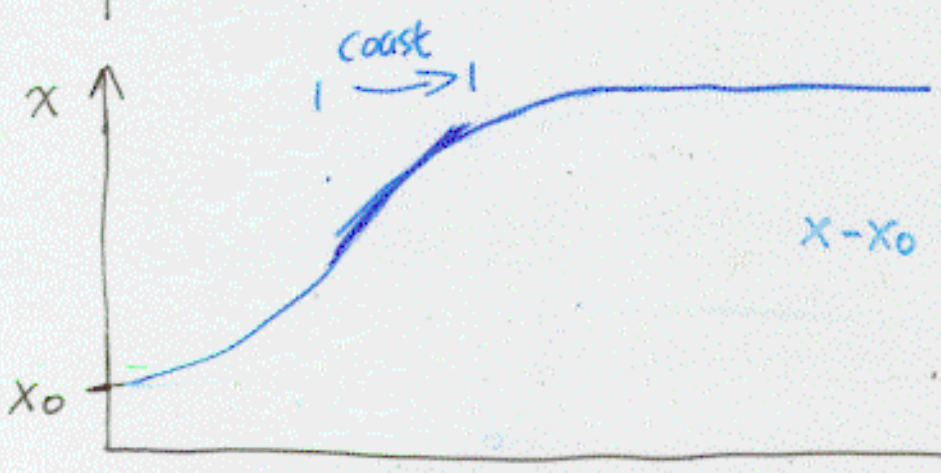
If speed v changes with time and $v(t) \equiv \frac{dx}{dt}$ (m/s)

Then position (displaced from x=0) $x(t) = \int v\,dt +$ const

const determined by conditions e.g. at t=0

So in general, to find change in position over time interval (t_1, t_2) – just find area under curve of v(t):





$$x - x_0 = \int_{t_0}^{t} v(t)\,dt$$

∴ If we use speedometer + watch to record v(t)
⇒ can find distance traveled $x - x_0$: no odometer needed

Position from Speed by Integration cont/d.

If position at $t=0$ is $x(0)$

From before: $x - x(0) = \int_{t=0}^{t=t} v(t)\,dt$

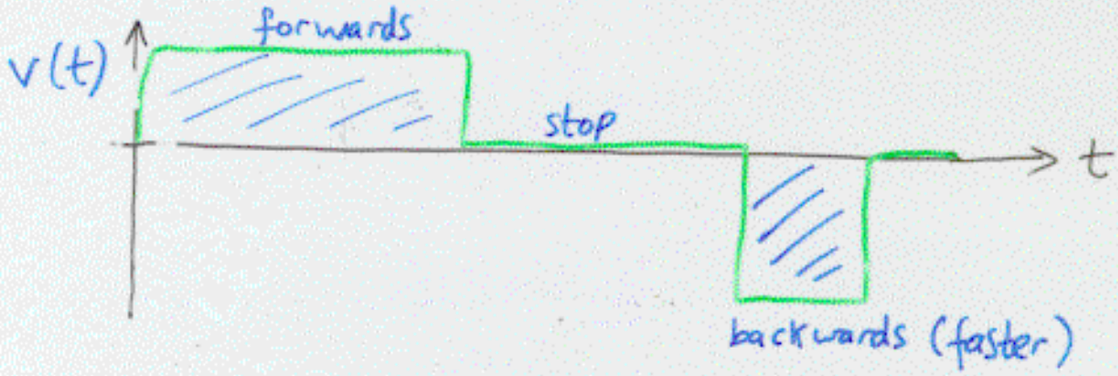
Check: For uniform speed, $v(t) = v$ (constant)

So $\int_0^t v\,dt = vt \Rightarrow x - x(0) = vt$

or $x = x(0) + vt$ ✓

Note: x is position w.r.t. origin, not path length

e.g. our round-trip to mailbox and back



Since we return to starting point, $x(t) - x(0) = \int v\,dt = 0$

i.e. area above time axis = area below

true for any round trip.

Example in 1-D (cf. Hecht example 2.5)

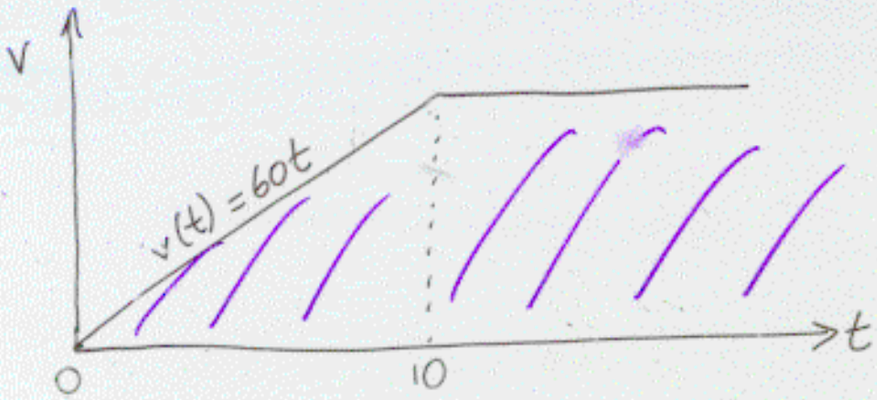
You are on a space station. At $t=0$, a probe 3m behind you fires its rocket such that $v(t) = 60t$ m/s for 10s.

What is:

a) Time and speed when rocket passes you ($x=0$)?

b) Distance + speed of rocket at $t=10s$?

c) Distance to rocket for times $t \geq 10s$?

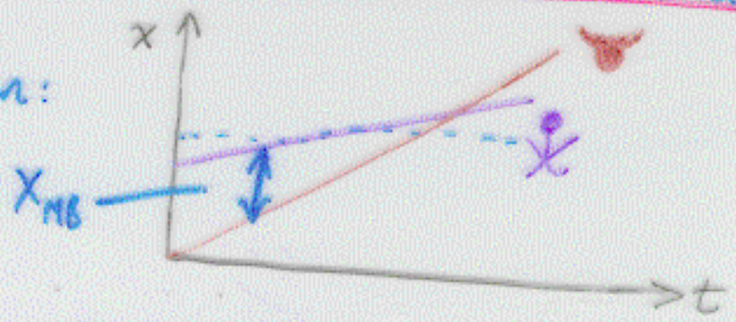


Given $v(t) = 60t$ and condition $x(0) = -3m$

$$x(t) = \int v(t)\,.\,dt + \text{constant}$$

Relative Motion in 1-D : Speeds add or subtract

e.g. Bull chases man:



Bull's position : $x_B = x_B(0) + v_B t$

Man's $x_M = x_M(0) + v_M t$

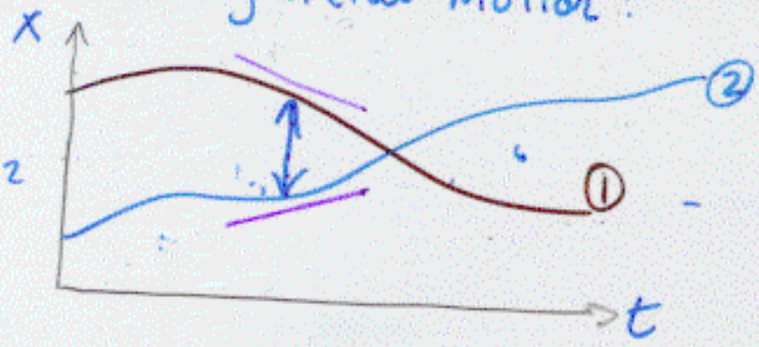
Now define x_{MB} = relative dist. of man from bull

i.e. $x_{MB} = x_M - x_B = [x_M(0) - x_B(0)] + [v_M - v_B]\,t$

initial separation "closing speed"

i.e. relative speed $v_{MB} = (v_M - v_B)$ – makes sense!

For more general motion:



At any instant

$v_1 = \frac{dx_1}{dt}$, $v_2 = \frac{dx_2}{dt}$

$\Rightarrow$ relative speed $v_{12} = v_1 - v_2$

(note: v_1, v_2 can be >0 or <0)

1-d example: relative speed algebra

Q. A train passes through station at 16 m/s
Passenger walks towards back of train at 2 m/s
– how fast are they moving relative to station?
And vice versa?

A We have $V_{ts} = +16\,m/s$, $V_{pt} = -2\,m/s$

$$\therefore V_{ps} = V_{pt} + V_{ts} = -2+16 = 14\,m/s$$

Note that $V_{pt} = -V_{tp}$ etc.

So passenger sees station pass by at:

$$V_{sp} = V_{st} + V_{tp}$$
$$= -16 + (-(-2)) = -14\,m/s = -V_{ps}$$