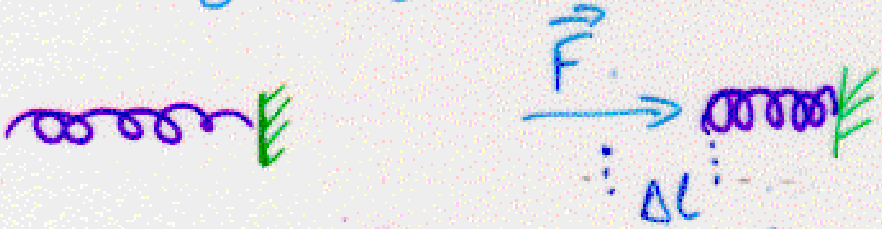
# Forces on Solids – Elasticity

Many Solids are compressible – atomic spacing ↓ as compressive force ↑. (Some can also be stretched)

Hooke's Law : For wires, bars, springs

Deformation (length change $\Delta l$) $\propto$ applied force

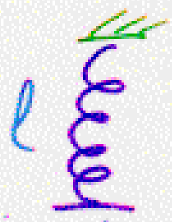


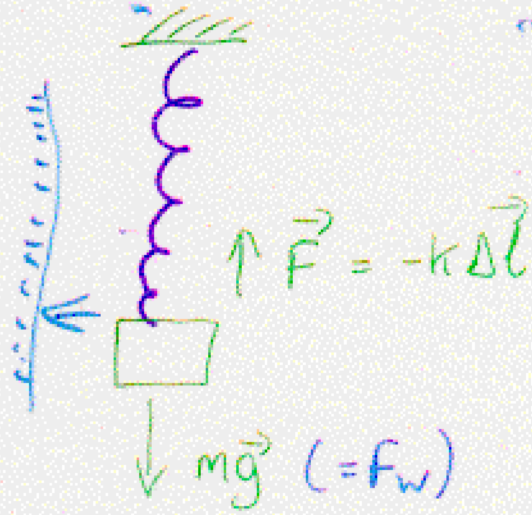

i.e. $\underline{\vec{F} = -k\,\Delta\vec{l}}$

$k$ = Spring constant (N/m or kg/s²)

$k$ is large for "stiff" spring

e.g. Spring balance to measure weight




unloaded $\Delta l = 0$

loaded : $k\Delta l = mg$

$\Rightarrow$ extension $\Delta l = \dfrac{mg}{k}$

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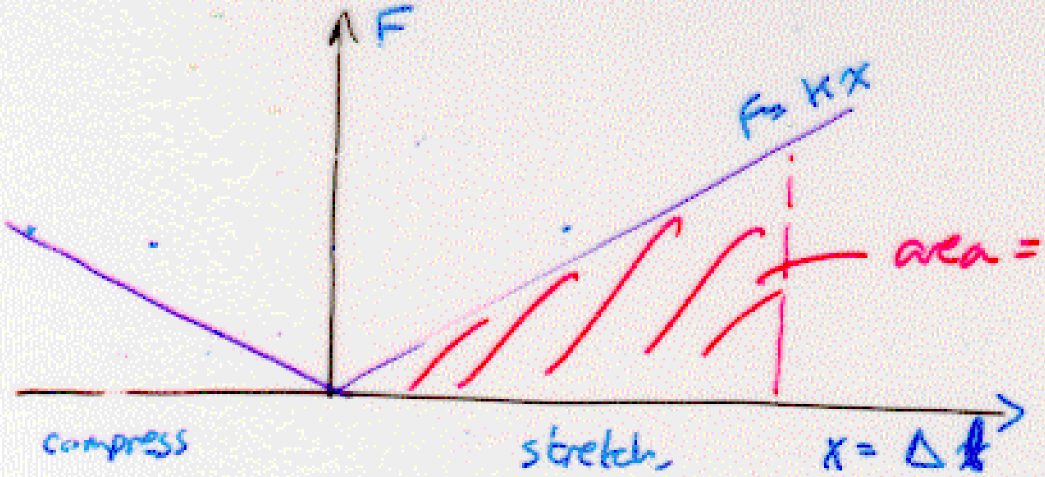
# Hooke's Law and Potential Energy

e.g. Spring (elastic) bumpers on air track convert K.e. to elastic p.e., then back again on rebound



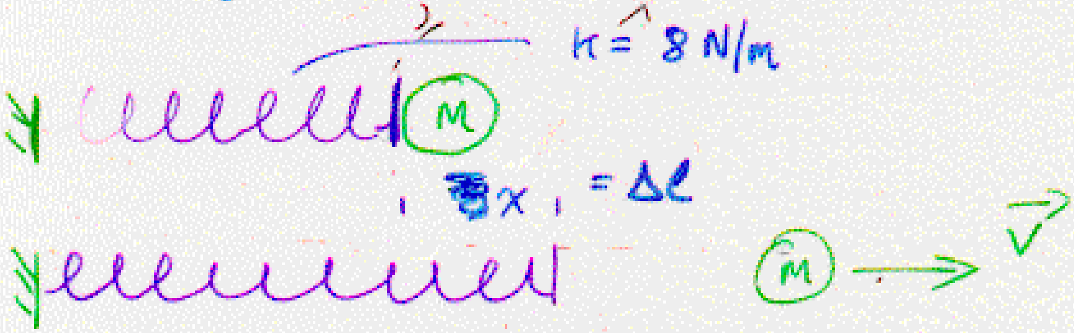

Work done to stretch/compress spring by distance x

$$W = \int_0^{\pm x} F.\,dx = \int_0^{\pm x} \underbrace{kx}\,.\,dx = \tfrac{1}{2}kx^2$$

= area under Δ on F vs. x plot (½ base × height)

e.g. Spring in pinball machine has k = 8 N/m

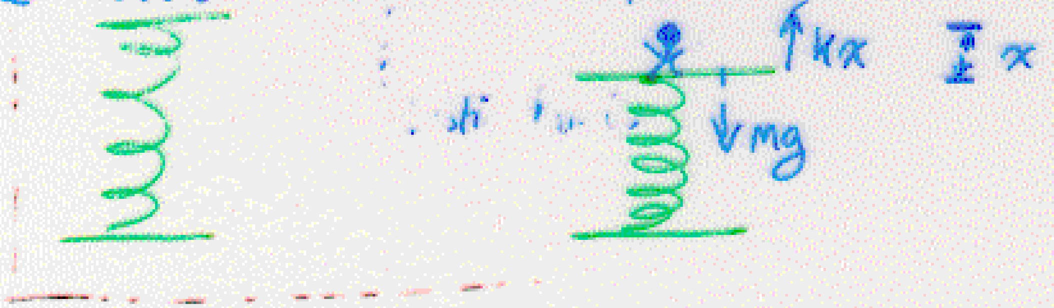
compress by $\Delta l = 0.04m$, what is speed of 20g ball?




KE gained $\frac{1}{2}mv^2$ = P.E lost = $\frac{1}{2}kx^2$

$\Rightarrow$ speed $v = \sqrt{\frac{k}{m}}\,.\,x \quad \sqrt{\frac{8}{.02kg}} \times 0.04 = 0.8\,m/s$

Person with $F_W = mg = 500$ N steps onto spring scale with $k = 2000$ N/m




Compression force $F = kx = mg$

$\Rightarrow$ person drops $x = mg/k = \frac{500}{2000} = 0.25$ m

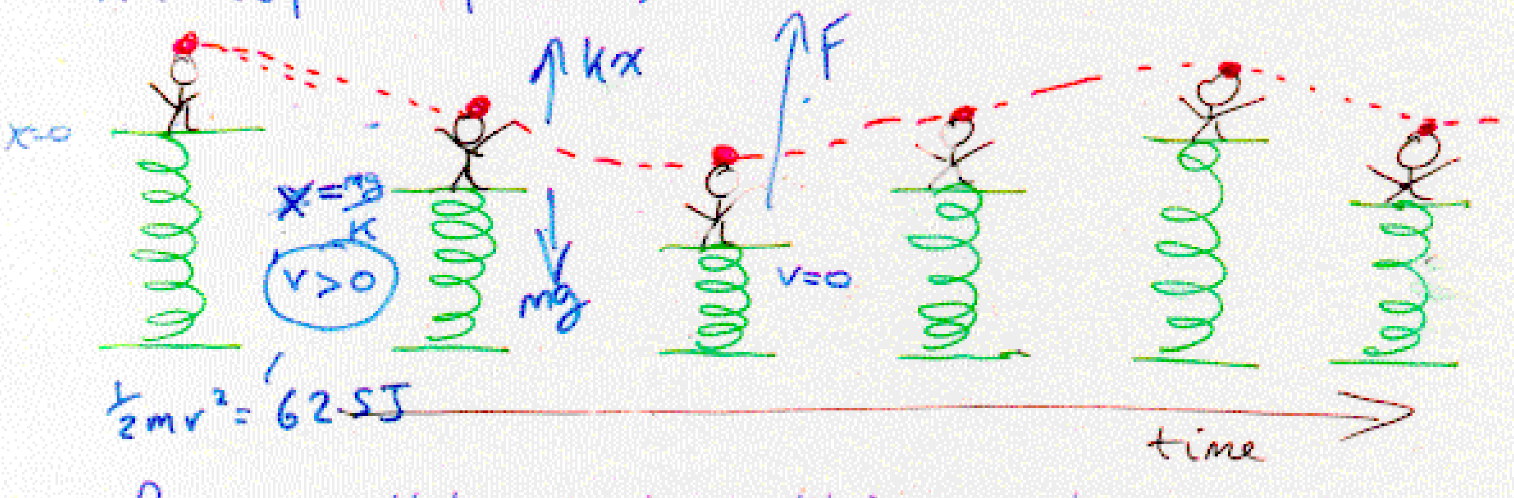
Grav. PE lost $= mg\,x = 500 \times 0.25$ m $= 125$ J

Elastic P.E. gained $= \frac{1}{2} k x^2 = \frac{1}{2} \cdot 2000 \cdot (0.25)^2 = \underline{62.5 \text{ J}}$

i.e. only 50% of the lost grav P.E.

Q What happened to the other 62.5 J of energy?

A (If no friction) .... MOTION!




Person oscillates about equilibrium position where $kx = mg$

Total energy $E = \frac{1}{2} k x^2 + \frac{1}{2} M V^2 = 125$ J.

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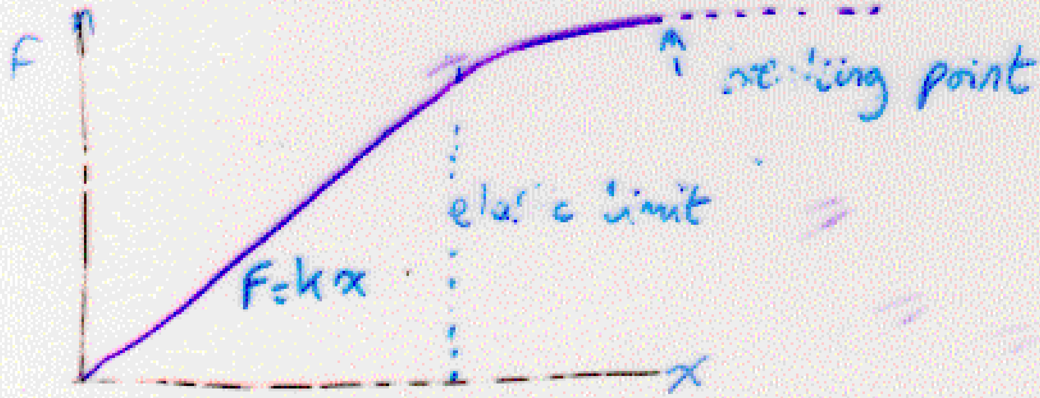
# Strength and Elastic Limit



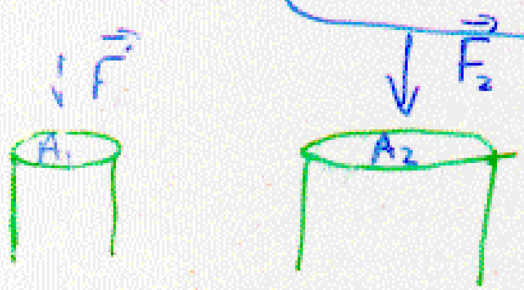

Incr... [illegible] (applied force compressing or stretching)

→ eventually spring will stretch permanently and then break

or for compression (e.g. granite, bone), substance will crumble under strain.

Increase area ⇒ distributes force so max. force ↑

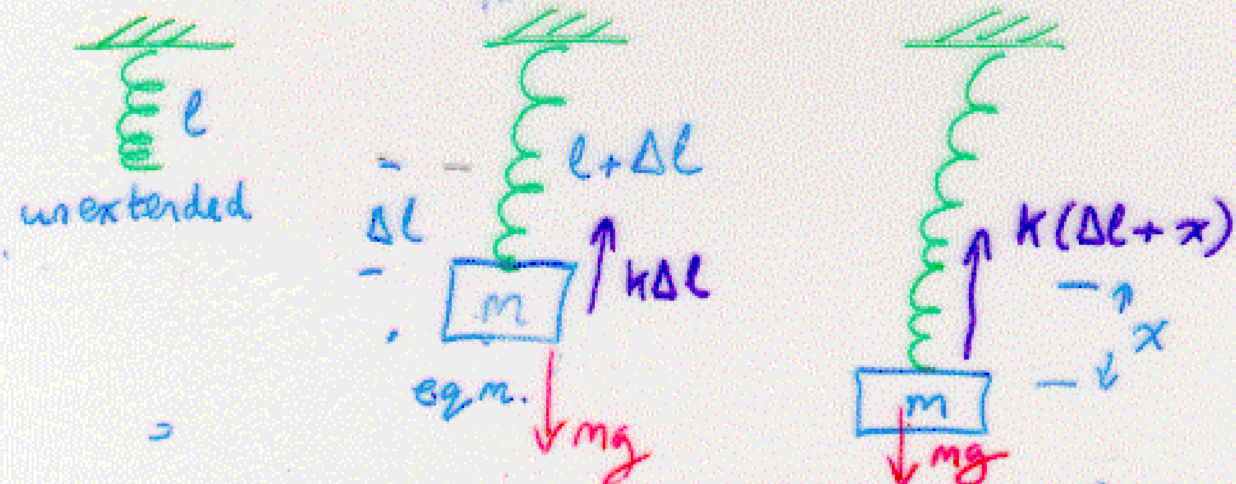
But define $\text{strain} = \dfrac{\text{force}}{\text{area}}$ (N/m² or Pa)




Find strength = maximum strain ~ constant for any material

(see Table 10·1)

# Motion of Mass on Spring




Hang mass from spring → new eqm. length $\ell + \Delta\ell$

such that $k\Delta\ell = mg$.

Now displace mass further by $x$

⇒ net restoring force $F = -kx$ (unbalanced)

From Newton II : $F = \boxed{m\dfrac{d^2x}{dt^2} = -kx}$ (*)

Solve this Equation of Motion to find $x(t)$

Try: $x = A\cos(\omega t + \phi)$

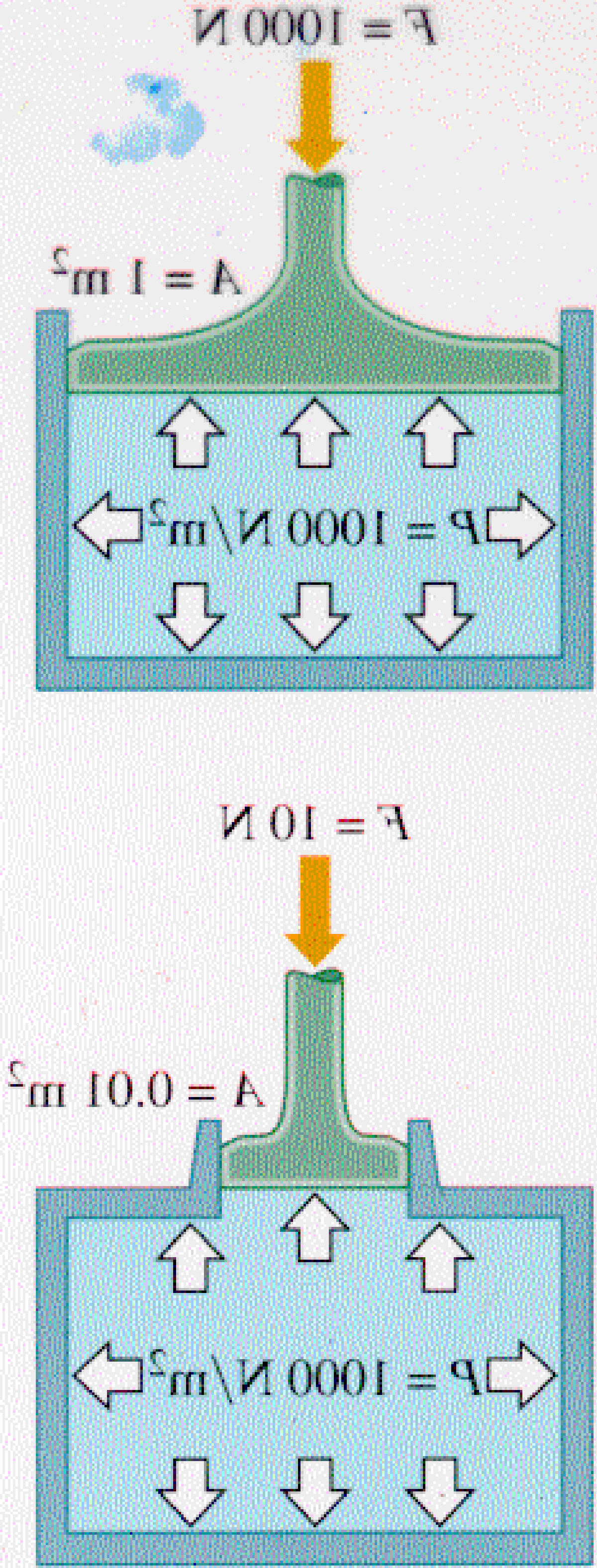
Then speed $v = \dfrac{dx}{dt} = -\omega A \sin(\omega t + \phi)$

Accel. $a = \dfrac{d^2x}{dt^2} = -\omega^2 \underbrace{A\cos(\omega t + \phi)} = \underline{-\omega^2 x}$

So (*) satisfied if $\underline{x = A\cos(\omega t + \phi)}$ with $\omega^2 = \dfrac{k}{m}$

Figure 9.16

# Determining piston pressure



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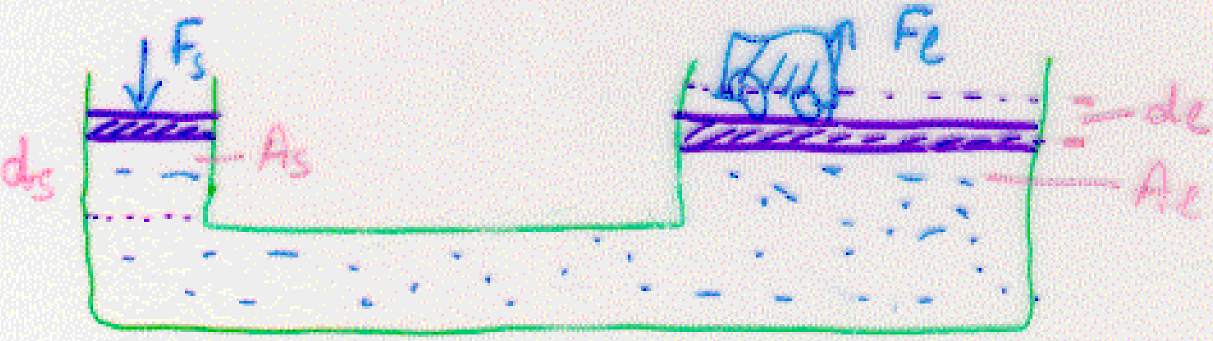
# Pascal's Principle : Hydraulic Lever




Push down on small piston with force $F_s$

$\Rightarrow$ increase in pressure $\Delta P = F_s/A_s$ at all points in fluid (Pascal's Principle)

$\therefore$ Net force up on large piston $F_\ell = \Delta P . A_\ell = F_s \, A_\ell / A_s$

i.e. fluid acts as force multiplier or hydraulic

What about work? If small piston moves down $d_s$

Work $= F_s d_s$ , volume displaced $= A_s d_s$

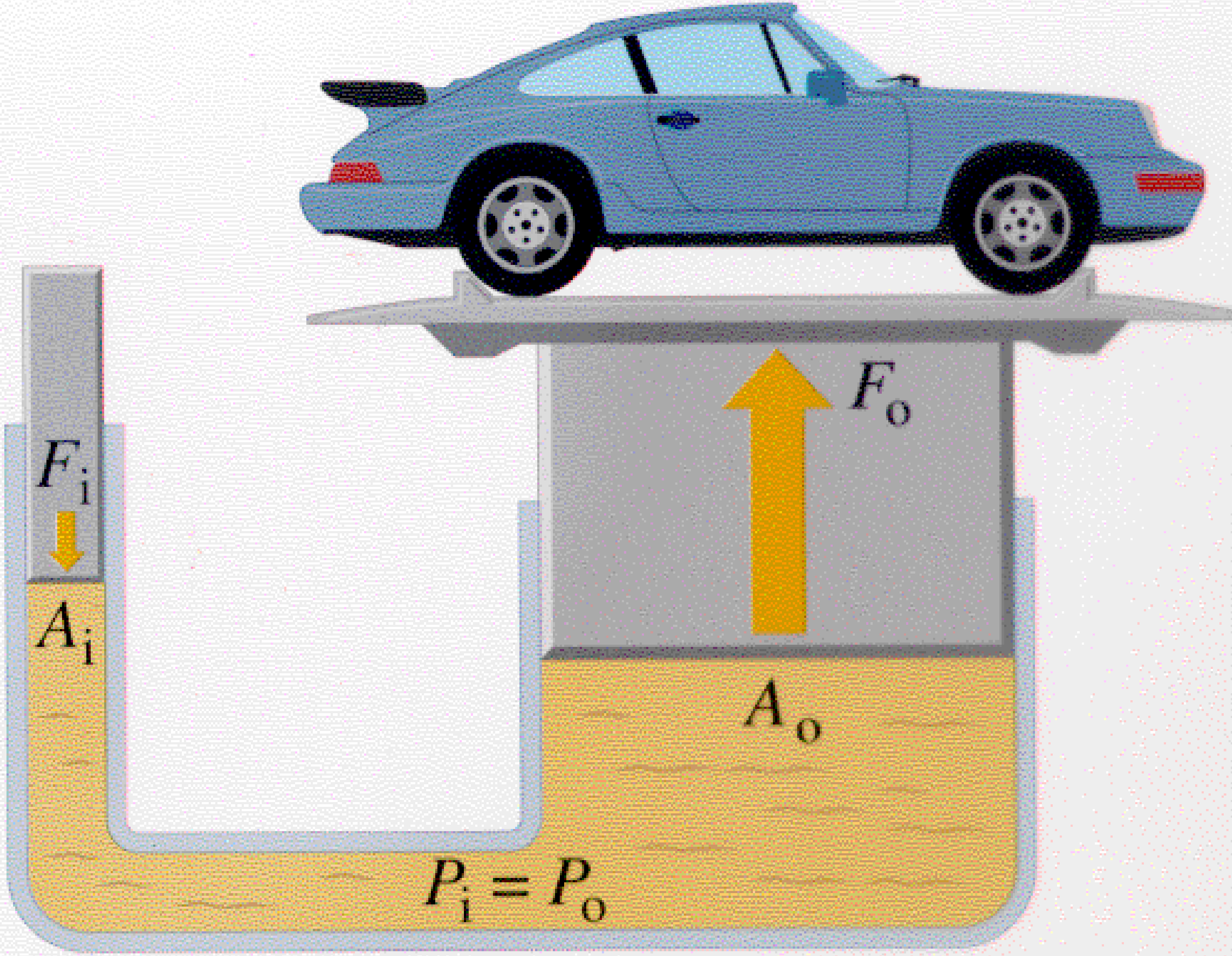
$\therefore$ large piston moves up by $d_\ell = \dfrac{\Delta V}{A_\ell} = \dfrac{A_s}{A_\ell} d_s$

$\therefore$ large piston does work $F_\ell d_\ell = \underline{\dfrac{A_\ell}{A_s}} F_s . \underline{\dfrac{A_s}{A_\ell}} d_s$

i.e. $W = F_\ell d_\ell = F_s d_s$ , energy conserved.

e.g. Can lift car with $mg = 10^4$ N using one hand

Figure 9.17
## A hydraulic lift

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