Hydrostatic Pressure

$$\frac{dP}{dh} = \rho g \ ^{(Pa/m)} \text{ : hydrostatic eqm.}$$

i.e. $P = \int_0^h \rho g \, dh$ in general

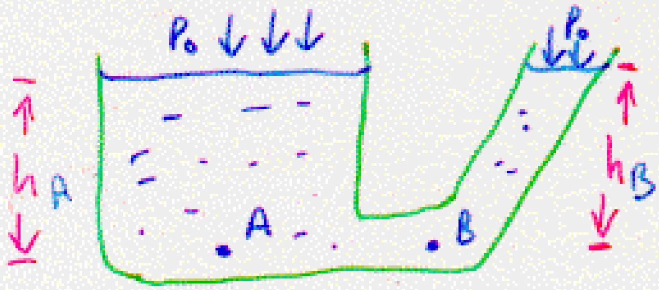
Meaning: Pressure (force/unit area) at depth h = weight of all material above that depth.

Pressure in Liquids : most have $\rho \approx$ constant ("incompressible")

Then $P = \rho g \int_0^h dh = P_0 + \rho g h$

$\uparrow$ pressure at $h=0$ (surface)

e.g. teapot



A, B at same depth

Pressure at A, B must be equal for eqm. (otherwise liquid forced from A $\Leftrightarrow$ B)

i.e. $P_A = P_B = P_0 + \rho g h$

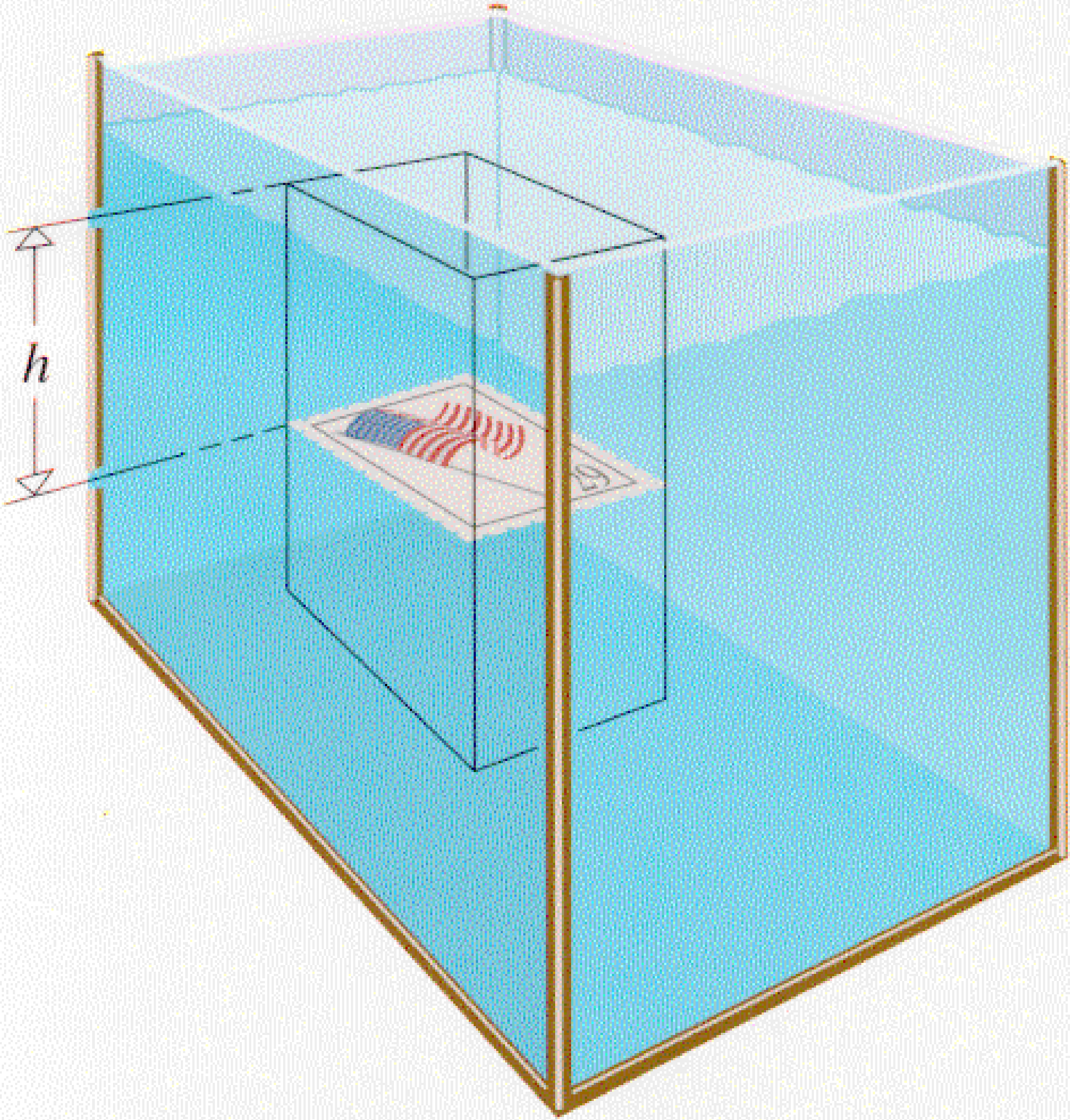
$\uparrow$ depends on depth only

$\Rightarrow$ depth h must be same for both $h_A = h_B = h$.

$\therefore$ "A liquid finds its own level"

Figure 9.4

Determining pressure of a liquid column



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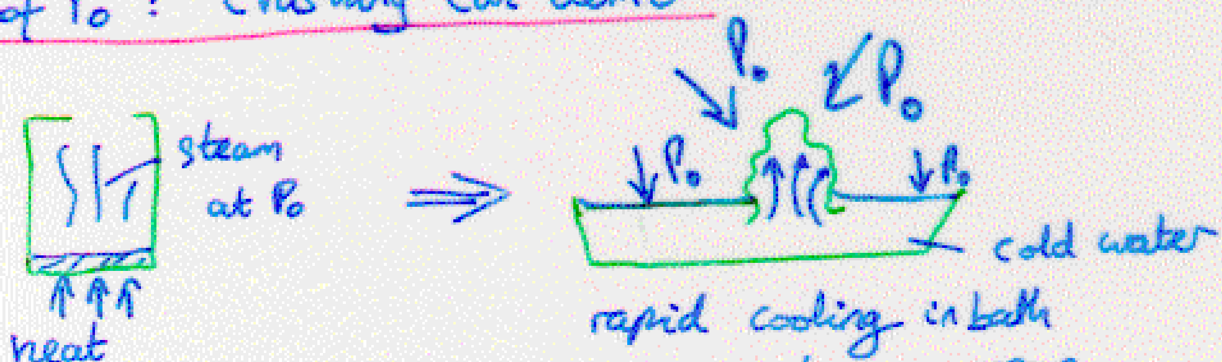
Atmospheric Pressure

At sea level $P_0 = \int_0^h \rho g \, dh$ = weight of air column (~300km) above unit area

$P_0 \simeq 10^5$ Pa (~16lb/sq.in)

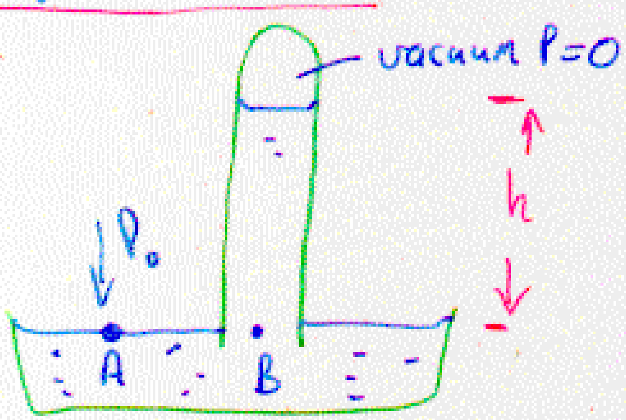
Effect of P_0 : Crushing Can demo



rapid cooling in bath reduces steam pressure

Result ① Can crushed by steam, ② atm. pressure pushes water into can to fill partial vacuum.

Liquid Barometer:



Pressure at level of surface

$P_A = P_B = P_0 = \rho g h$

For mercury, $P_0 = 10^5$ Pa

$\Rightarrow h = 0.76$m

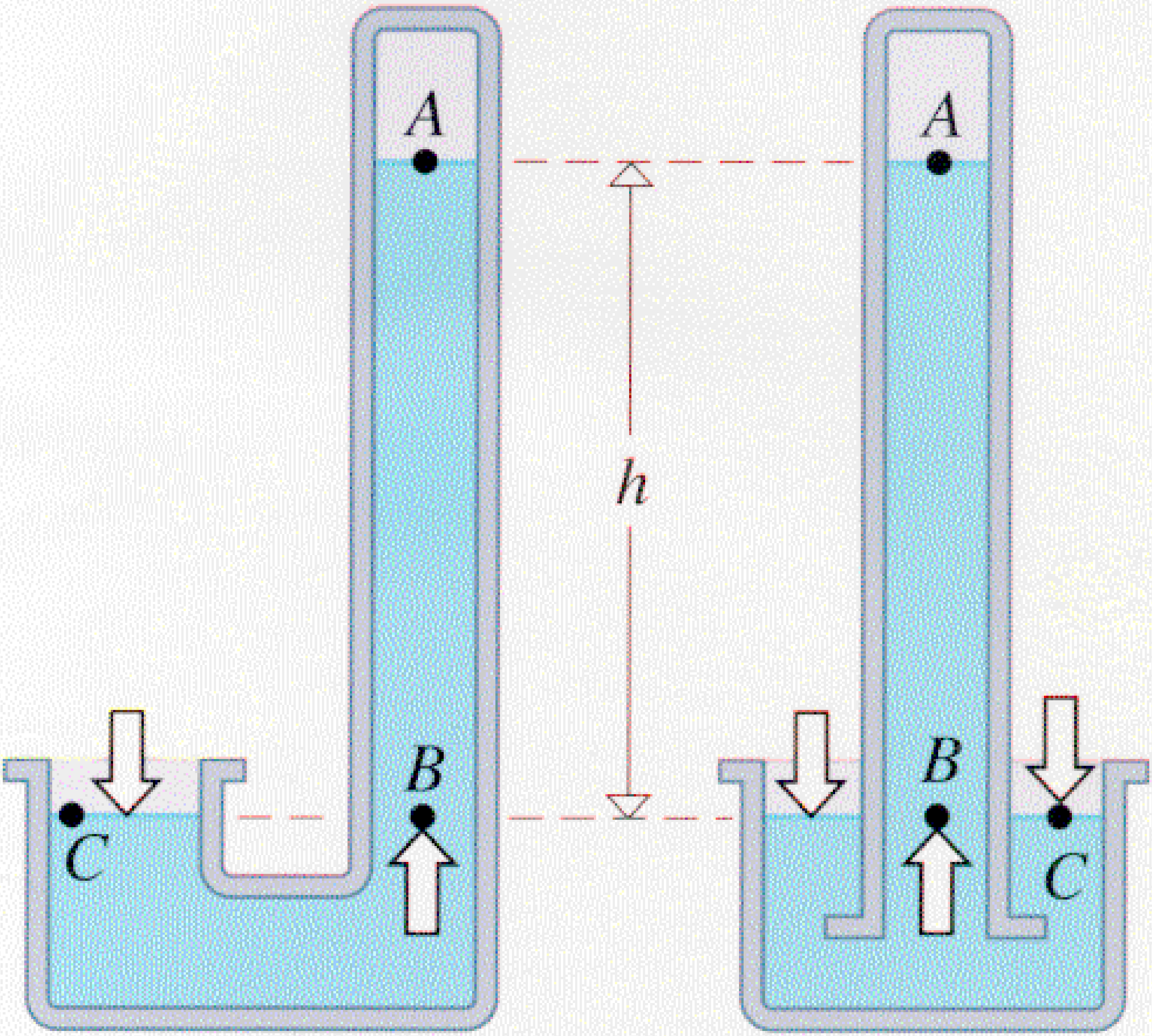
For water $\Rightarrow h \simeq 10$m (30ft)!

Water also produces vapor, destroying vacuum.

We can use a siphon to lift water up to 10m at sea level

Figure 9.11

Two equivalent versions of the barometer



Hydrostatic Pressure - Examples

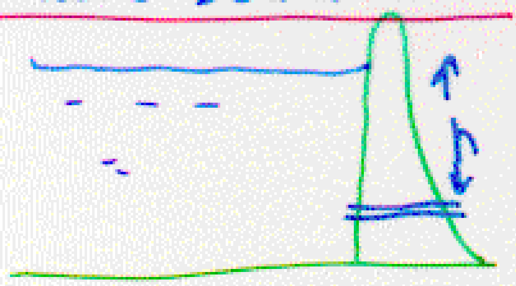
1. For human ~2m tall, $\rho_{blood} \sim 1.1 \times 10^3\ kg/m^3$

On earth ($g = 10$) $\quad \Delta P$ (head → feet) $= \rho g h \sim 10^4 Pa$

In free-fall ($g = 0$) $\quad \Delta P = 0 \quad$ (light-headed)

Upside-down $\quad \Delta P = -10^4 Pa \quad$ (dizziness, red-out)

2. Pressure behind dam



P depends on depth only, not volume.

So force on thumb-sized hole ($A = 10^{-4} m^2$) at depth h

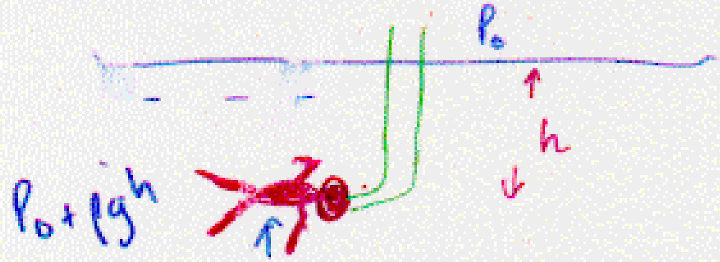
$$F = P.A = (\underbrace{P_0}_{10^5 Pa} + \underbrace{\rho g h}_{= 5\times10^4 Pa \text{ at } 5m \text{ depth}}) \times 10^{-4} m^2 = 15\ N.$$

3. Snorkeling depth. Snorkel connects lungs to $P = P_0$

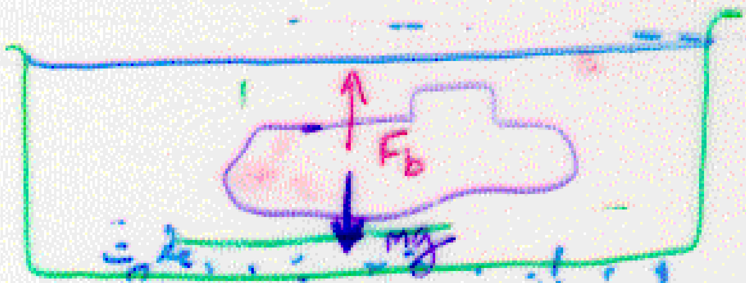
Pressure on diver's chest $P = P_0 + \rho g h$

∴ lungs/chest wall must withstand $\Delta P = \rho g h$

(For depth $h \gtrsim 0.5m$, ΔP collapses lungs)



Hydrostatic Pressure + Buoyancy: Archimedes



Draw boundary around a piece of liquid

At eqm. weight of fluid inside mg is balanced by pressure differences above + below

i.e. Buoyant force $F_b = \int P.dA = mg = \rho V g$

Now <u>remove</u> liquid from inside boundary and replace with object (e.g. sub) mass M

Buoyant force still provided by surrounding fluid

$\Rightarrow$ net force $= Mg - \overbrace{\rho V g}^{F_b}$ = "effective weight"

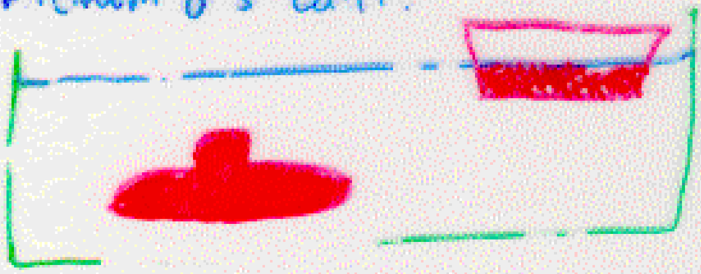
"A body immersed in fluid experiences a buoyant force F_b = weight of water displaced." $F_b = \rho V g$

$\therefore$ Net force downwards $Mg - \rho V g$ >0 : sinks

<0 : floats

$=0$: stable

• Archimedes centr.



• Effective weight of submerged object $= Mg - \rho V g$

→ to simulate reduced gravity
(bottom of swimming pool = moon)

Or if $M < \rho V$ i.e. $\rho_{obj} = \frac{M}{V} < \rho$,

object pushed up out of water until it floats

~~the same~~ s.t. $\underline{\rho V_{subm} g = Mg}$

e.g. ships with steel hulls, icebergs etc.

e.g. For iceberg with mass M, $\rho_{obj} = 900\,kg/m^3$

$$\rho V_{sub} \cancel{g} = M\cancel{g} = \rho_{obj} V g$$

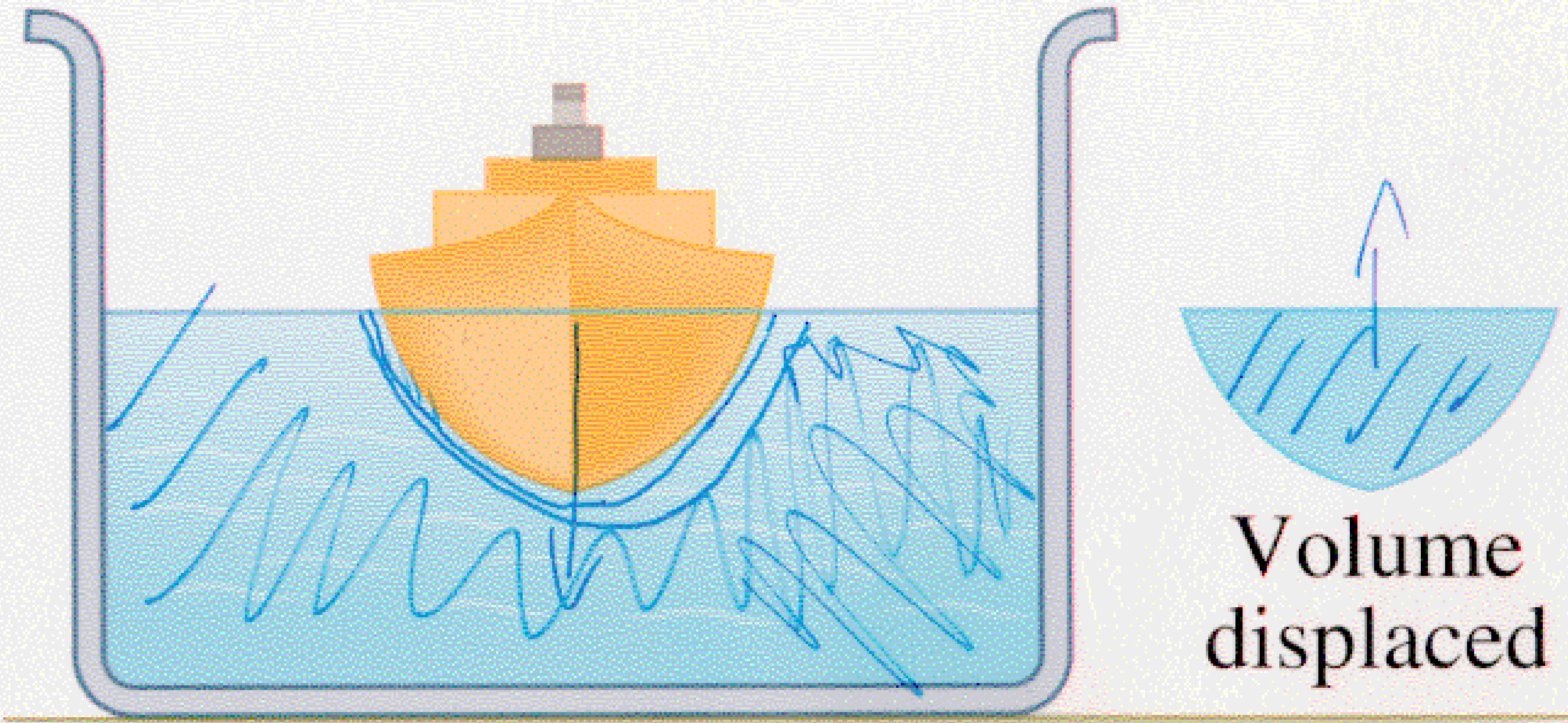
$$\Rightarrow \frac{V_{subm}}{V} = \frac{\rho_{obj}}{\rho} = \frac{900}{1000}$$ or 9/10 volume underwater

(i.e fraction $\underline{1 - 9/10 = 1/10}$ is above surface)

c.f. floating person has $\frac{V_{subm}}{V} \approx 0.97 \Rightarrow 3\%$ above surface

Figure 9.20

A floating object displacing liquid



$Mg = \rho V g \, (F_B)$