Rotational Equilibrium + Torque

e.g. Exert equal but opposite forces each side of steering wheel:

$\vec{F}_L = -\vec{F}_R$

No net force? ✓ So no net motion? ✗

$\vec{F}_L + \vec{F}_R = 0$

Wheel starts to rotate, $\alpha = \frac{d\omega}{dt} \neq 0$

(though wheel does not translate)

Reason? $\vec{F}_L, \vec{F}_R$ do not act at the same point of application.

$\therefore$ Both $\vec{F}_L, \vec{F}_R$ provide a TORQUE ("twist") which does not cancel.

Define Torque (or moment) of force about a rotation center as: $\tau = r F_t = r F \sin\theta$
(pivot)

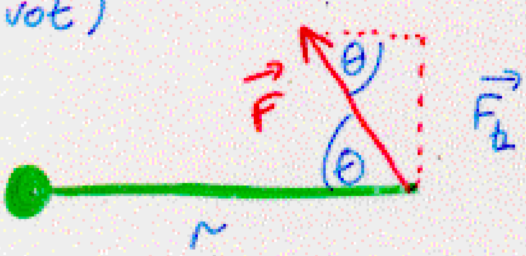
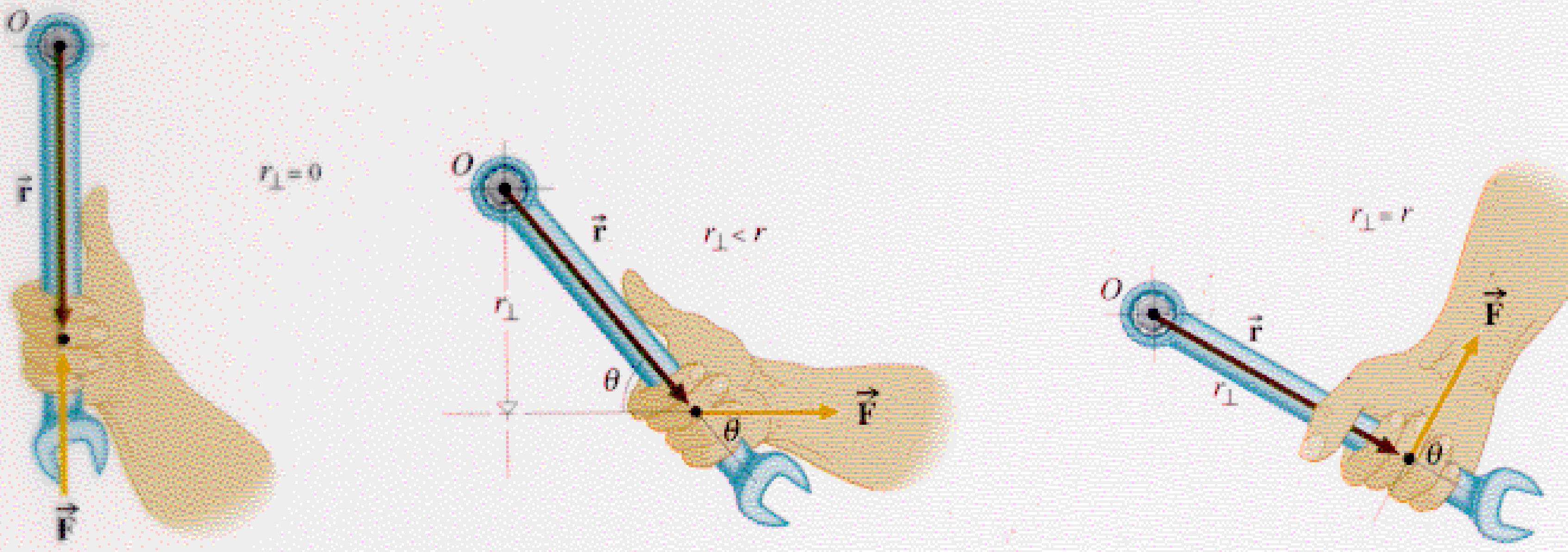


Figure 8.14

Torque $= rF\sin\theta = r_\perp F\sin\theta = rF_\perp \sin\theta$



(a) $\tau = 0$

(b) $\tau = rF\sin\theta$

(c) $\tau = rF$ $(\sin\theta = 1)$

21/3

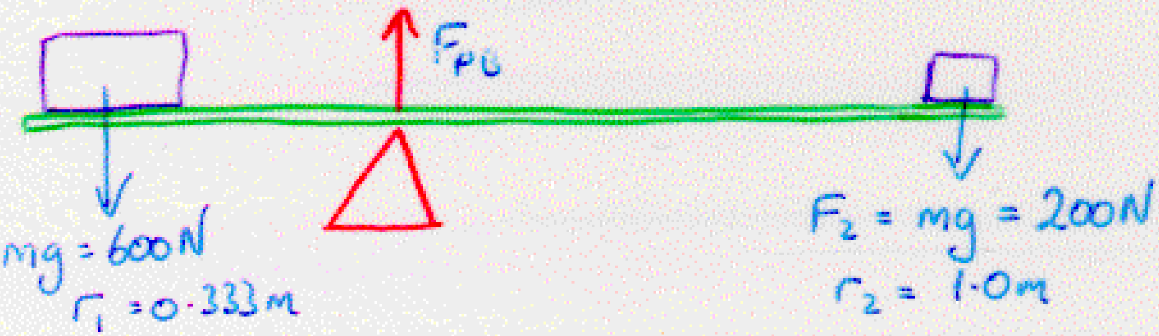
Rotational Eqm. cont/d.

So if $\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots = 0$: no linear acceleration

But only if $\sum \tau = F_1 r_1 \sin\theta_1 + F_2 r_2 \sin\theta_2 + \ldots = 0$ *

do we get no angular acceleration.

E.g. Balance weights, beam on pivot.



For "balance" ($\omega = 0$, $\alpha = \frac{d\omega}{dt} = 0$)

choose anti-clockwise as positive direction for ω $\quad F_1 r_1 + F_2 r_2 = 0$

Then torques $F_1 r_1 = -F_2 r_2 = 600 \times 0.333m$

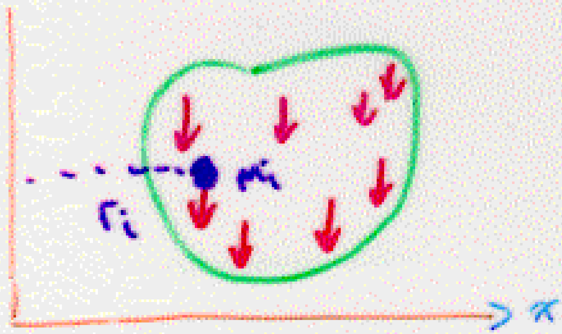
$= 200$ Nm.

Note: $F_1 + F_2 = 600 + 200N \neq 0$, but additional force of pivot on beam $F_{PB} = 800N$ in opposite direction

(and since $r = 0$, this force exerts no torque)

$\Rightarrow$ no net translation <u>OR</u> rotation.

Center of Gravity Extended Objects



Hold irregular object in outstretched hand

To calculate torque, can add up $\sum m_i r_i$

Easier to define center of gravity: point through which total weight can be thought to act.

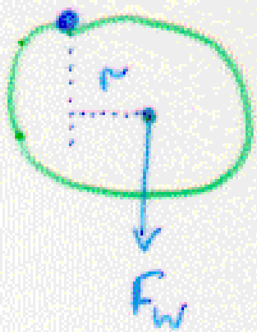
i.e. Torque $\tau = \sum m_i g x_i \equiv M g x_c$ where $M = \sum m_i$

i.e. $$x_c = \frac{\sum m_i x_i}{\sum m_i}$$

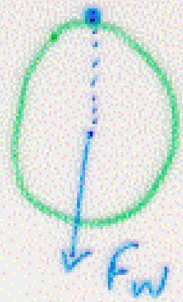
Similarly for y_c, z_c in 3-dimensions

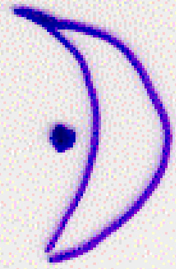
Center of Gravity and Stability

1. Hang object from one edge and let go:



$\tau = r_\perp F \neq 0$



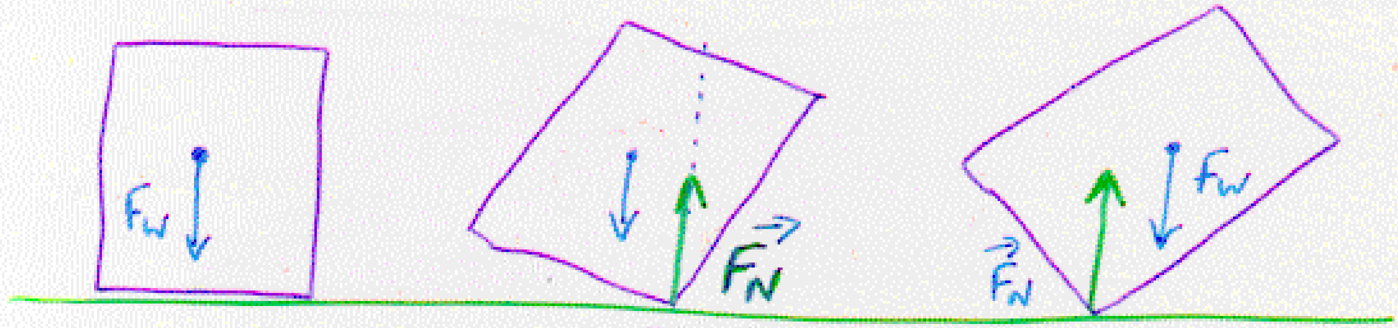


Net torque rotates object .... until $r \parallel F$ and $\tau = 0$

⇒ easy method to locate c.g. of object

Note: C.g. not always located inside object, e.g. crescent or banana shape

2. Rest object on horizontal surface, then try to tip over (to right):



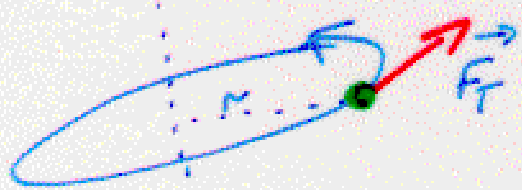
As long as c.g. stays to left of pivot, torque $\tau = r_\perp F_W$ acts to restore equilibrium ($\vec{F}_N = -\vec{F}_W$ but exerts no torque about pivot)

Rotational Dynamics

Already have 1st law of R.D:

(1) A body will remain at rest or rotate with ω = constant unless acted on by external torque

Now go back to small mass moving in circle



Tangential accel $a_t = r\frac{d\omega}{dt} = r\alpha$ from before

Newton II : $F_t = ma_t = mr\alpha$

This force exerts a torque with $F_t \perp r$

$$\tau = F_t . r = mr^2\alpha$$

If we define Moment of Inertia $I = mr^2$

(2) $$\tau = I\alpha \quad (\text{c.f. } F = ma)$$

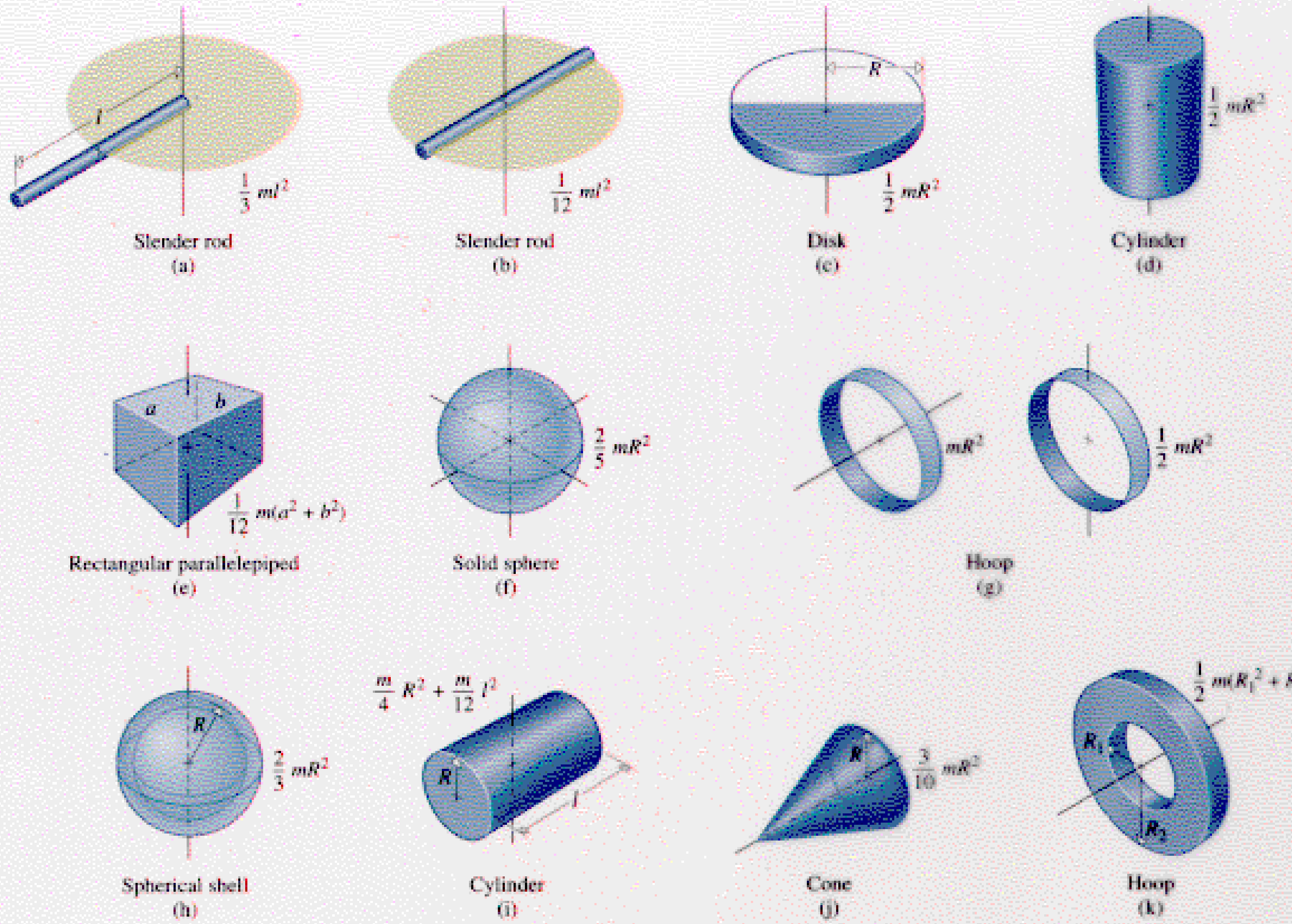
For many point masses in a rigid body (same ω, same α)

Total torque

$$\tau = \sum m_i r_i^2 . \alpha = I\alpha$$

where $I = \sum_i m_i r_i^2 = \int r^2 . dm$

Table 8.3
Moments of inertia



$I = \int r^2 . dm = \int r^2 \rho dV$. depends on location and direction of rotation axis