# Reading Quiz #6

1. The units of TORQUE are:

$Fr_{\perp} = Fr\sin\theta$

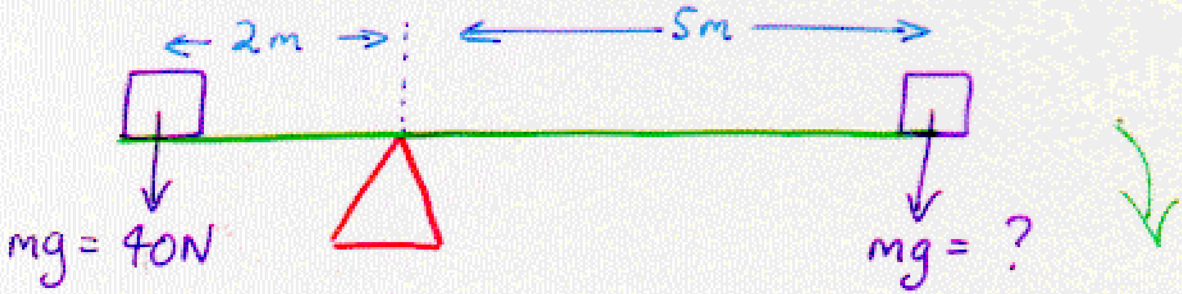
a) N/m

b) N m$^2$

c) N m

d) N m/s

2. For this balance to be in equilibrium, the second weight must be:




$\tau_L = 80\,Nm$

$\tau_R = 80\,Nm$

$(mg) \times 5m$

a) 16N

b) 40N

c) 100N

d) 200N

3. The Moment of Inertia of an object of given mass depends on its

a) Size and shape (mass distribution) ✓
b) Rotation axis ✓
c) Angular acceleration $\alpha$ ✗
d) A and B only
e) A, B, and C

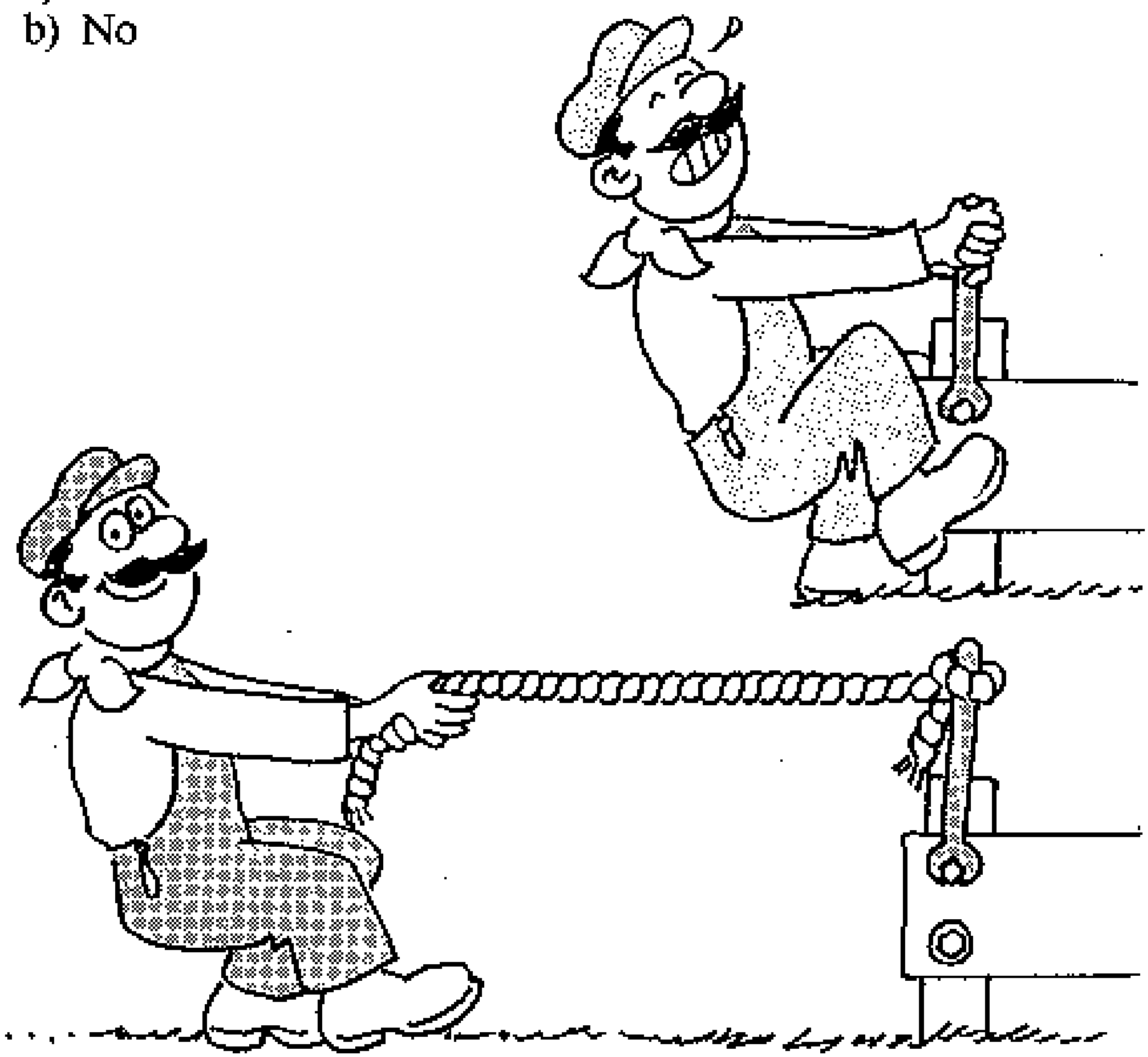
4. A spinning skater pulls in her arms and rotates faster because:

a) She produces a torque
b) the friction is reduced
c) angular momentum is conserved
d) none of the above.

## TORQUE

Harry is finding it very difficult to muster enough torque to twist the stubborn bolt with a wrench and he wishes he had a length of pipe to place over the wrench handle to increase his leverage. He has no pipe, but he does have some rope. Will torque be increased if he pulls just as hard on a length of rope tied to the wrench handle?

a) Yes
b) No



# Questions to Ponder .....

- Why do cars "nose up"/"nose down" when they accelerate / decelerate ?

- Why do single-rotor helicopters need a small vertical tail-rotor ?

- How does a long pole help a tightrope walker maintain their balance ?

- Why do some things tip over easily and others not? (e.g. wineglass vs. mug, Dolly Parton vs. ....?)

- Does a thrown hammer / tomahawk follow the same path if it is spinning ?

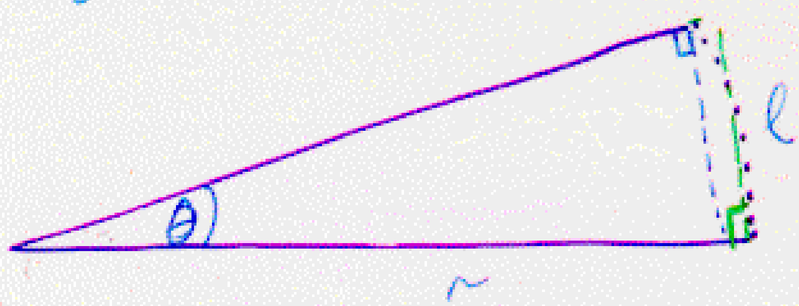
# Rotational Motion (Hecht ch. 8)

General displacement of finite, rigid body

= translation (body moves ∥ to itself
same disp. for each point)

\+ rotation through some angle $\theta$
about some center point.

For rotation angle $\theta$ (radian), at radius $r$




arc length $l = r\theta$ (definition of radian)

Note also if $\theta$ is small, $l \approx r\sin\theta$ and $l \approx r\tan\theta$

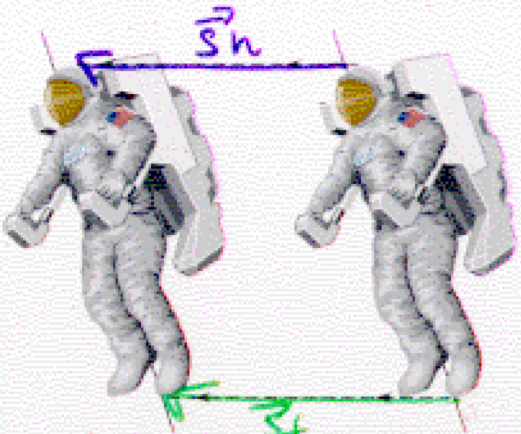
$\Rightarrow$ $\theta \approx \sin\theta \approx \tan\theta$ ($\theta$ in <u>radians</u>)

e.g. thumb at arm's length, moon, sun all have $\approx 1/2°$

$$\theta = \frac{l}{r} = \underset{\text{thumb}}{\frac{0.6\,\text{cm}}{70\,\text{cm}}} \approx \underset{\text{moon}}{\frac{3.5\times10^{6}\,\text{m}}{3.8\times10^{8}\,\text{m}}} \approx \underset{\text{sun}}{\frac{1.4\times10^{9}\,\text{m}}{1.5\times10^{11}\,\text{m}}} = 0.009\ \text{rad}$$

Figure 8.1

# Rotational and translational motion

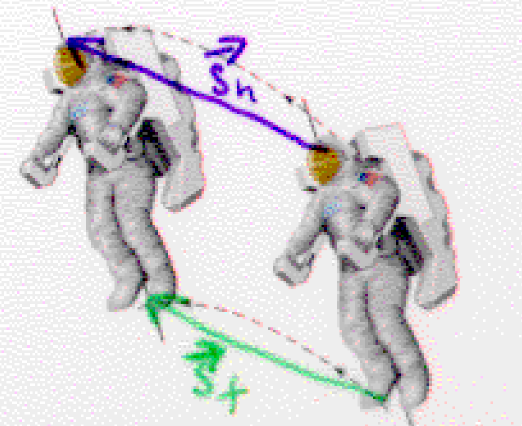



Rectilinear (along a straight line) translation

(a)

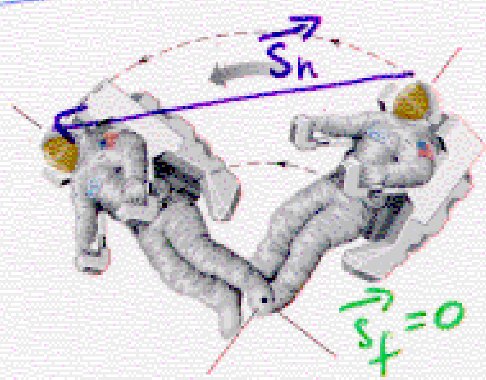
$\vec{s}_h = \vec{s}_f$

$\Rightarrow$ translation only




Curvilinear (along an arc) translation

(b)

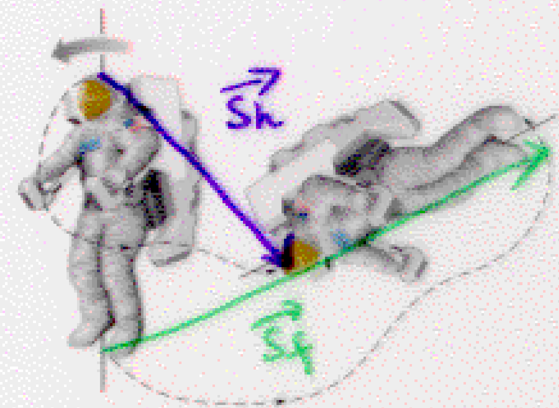



Rotation (about a point within the body)

(c)

$\vec{s}_f \neq \vec{s}_h$

$\Rightarrow$ rotation (+ translation)



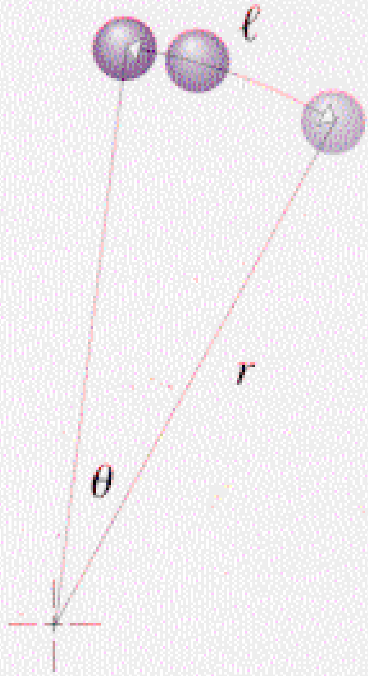

Rotation and translation

(d)

Figure 8.2

# Revolving string of beads sweep through the same angle

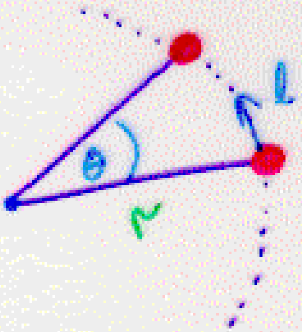

(a)




(b)

# Angular displacement and velocity

if cork rotates on string with period $T = 0.4\,s$




angular displacement $\theta$

$= 2\pi$ rad. per $0.4\,s$

$\Rightarrow$ Angular velocity $\omega = \dfrac{d\theta}{dt} = \dfrac{2\pi\,\text{rad}}{0.4\,s} = \dfrac{2\pi}{T}$

$= 5\pi$ rad/s or $15.71$ rad/s

Distance traveled $l = r\theta$ so

speed $v = \dfrac{dl}{dt} = r\dfrac{d\theta}{dt} = r\omega$ (m/s)

For $r = 0.5\,m$, $v = 0.5 \times 15.71 = 7.85$ m/s

So $\boxed{\omega = \dfrac{d\theta}{dt} \quad \text{and} \quad v = r\omega \quad (\vec{v} \text{ tangential to path, } \perp r)}$

$\therefore$ If $r, \omega$ constant, then $v$ is constant

and angle $\theta = \theta_0 + \omega t$.

# Circular Motion with Angular Acceleration

(e.g. Spin wheel or CD faster / slower)

If ang. speed $\omega$ changes,

define Ang. accel. $\alpha \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Then since $v = r\omega$, if $r$ constant $\frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$

tangential to $r$

i.e. $a_t = r\alpha$ : tangential accel.

Need to add the vector $\vec{a_t}$ to centripetal acel.

$$a_c = \frac{v^2}{r} \quad (m/s^2)$$

e.g. Car accelerates around $r = 50m$ bend such that $\frac{dv}{dt} = a_t = 10 m/s^2$ and $v = 30 m/s$ at that instant.

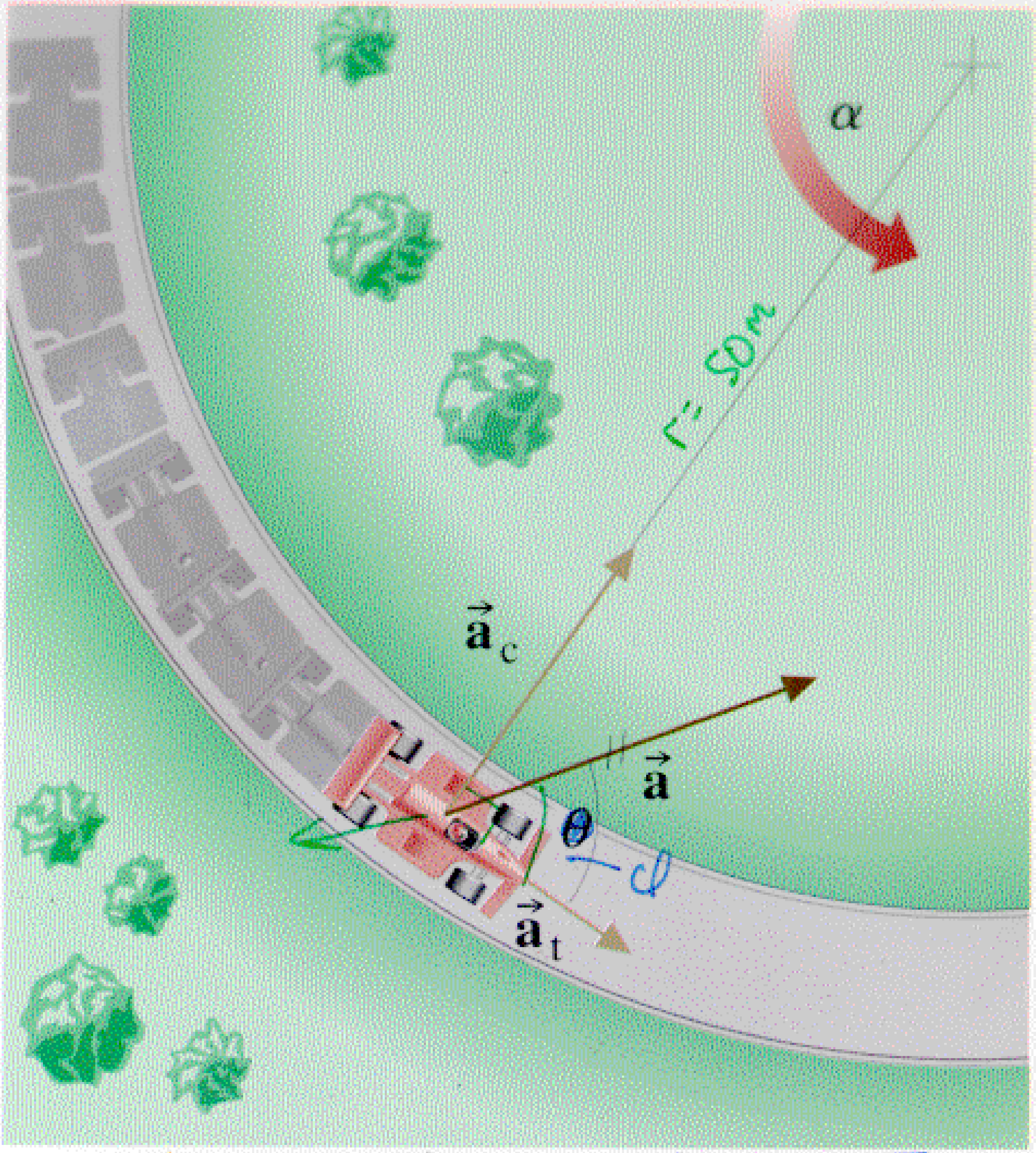
$\Rightarrow$ Ang. speed $\omega = \frac{v}{r} = 0.6$ rad/s, $\alpha = \frac{1}{r}\frac{dv}{dt} = 0.2$ rad/s$^2$

and Centripetal accel. $= \frac{v^2}{r} = \frac{(30^2)}{50} = 18 \, m/s^2$

(Ex. 8.4)

Figure 8.10

# Car around a curve: tangential and centripedal acceleration

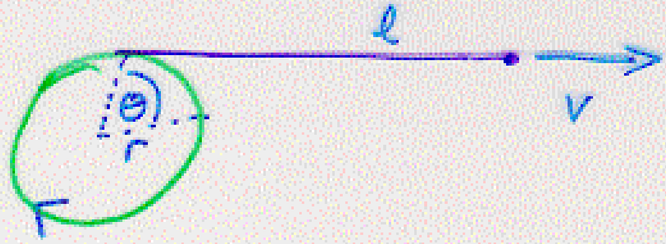



Total accel. $\left(= \frac{\text{force}}{\text{mass}}\right)$ $a = \sqrt{a_c^2 + a_t^2} = 21\,m/s^2$
$(\approx 2g)$

in direction $\phi = \tan^{-1}\frac{a_c}{a_t} = 61°$ from $\vec{v}$.

# Constant Angular Acceleration

e.g. Carpet loom starts up, pulling thread off spool such that ang. speed changes from 0 ($=\omega_0$) to $\omega$ in time $t$:



Since $v = r\omega$, $\frac{dv}{dt} = r\alpha$, can divide constant-accel linear eqns. by $r$ to give

$$\omega = \omega_0 + \alpha t \qquad v = v_0 + at$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad l = v_0 t + \tfrac{1}{2}at^2$$

$$\text{So} \quad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad v^2 = v_0^2 + 2al$$

e.g. An $r = 0.5$cm spool unwound from rest spins up to 120rpm in $t = 15$s. How much thread is unwound? How fast is thread traveling?

120 rpm $\Rightarrow$ $\omega = \frac{120 \times 2\pi \text{ rad}}{60\text{s}} = 4\pi$ rad/s

(period $T = \frac{2\pi}{\omega} = 0.5$s)

$\therefore$ Ang. accel $\alpha = \frac{\omega - \omega_0}{t} = \frac{4\pi - 0}{15\text{s}} = 4/15\,\pi$ rad/s$^2$

$\Rightarrow$ Angle turned $\theta = \omega_0 + \frac{1}{2}\alpha t^2 = \frac{\omega^2 - \omega_0^2}{2\alpha} = 30\pi$ rad

$\left(= \frac{30\pi}{2\pi} = 15 \text{ rotations}\right)$

$\Rightarrow$ Length $l = r\theta = \underset{cm}{0.5} \times 30\pi = 47.12\,cm$

Speed $v = r\dfrac{d\theta}{dt} = r\omega = 0.5 \times 4\pi\ rad/s = 6.28\,m/s$