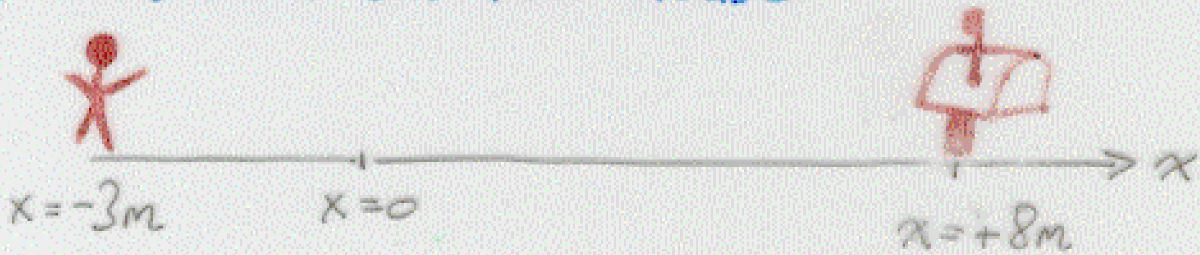
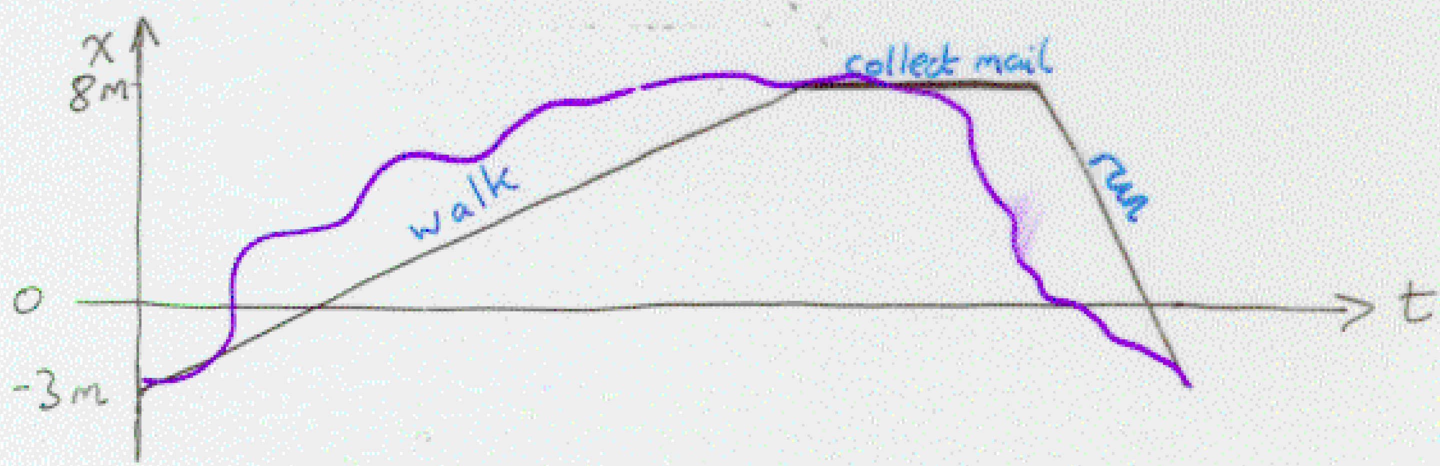
Position, Speed, Velocity (ch. 2)

E.g. A person walks across street to mailbox,
collects mail,
then runs back into their house



Can describe motion as $x(t)$:



Note: Many functions $x(t)$ possible,
but must be:

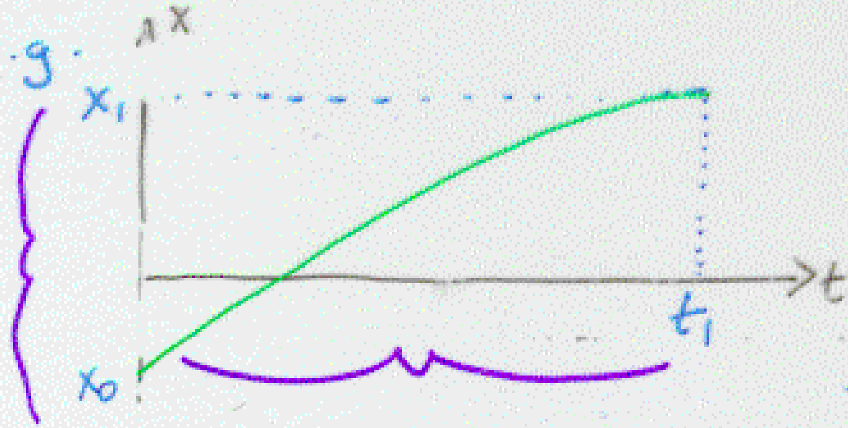
- continuous (no "teleportation")
- single-valued (no "time loops")

How fast? Average Speed

DEFINE: Average Speed $V_{av} \equiv \dfrac{\text{distance along path}}{\text{time taken}} \quad \dfrac{l}{t}$

Units: $\dfrac{\text{distance}}{\text{time}} = \dfrac{\text{meters}}{\text{seconds}} = m/s$ or ms^{-1}!

e.g.

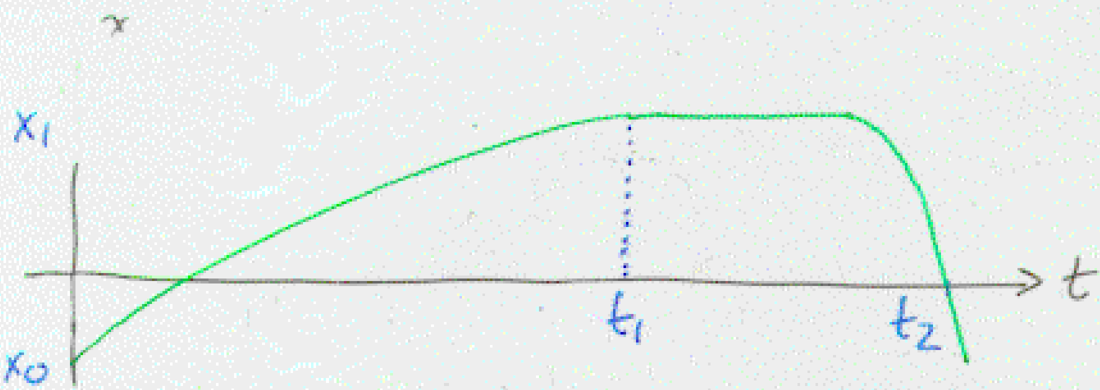


In time t_1, distance traveled $l = x_1 - x_0$

$\Rightarrow V_{av} = \dfrac{(x_1 - x_0)}{t_1}$

Note: path length l = "odometer reading"

be careful with round trips



Split into segments: path length $l = 2(x_1 - x_0)$

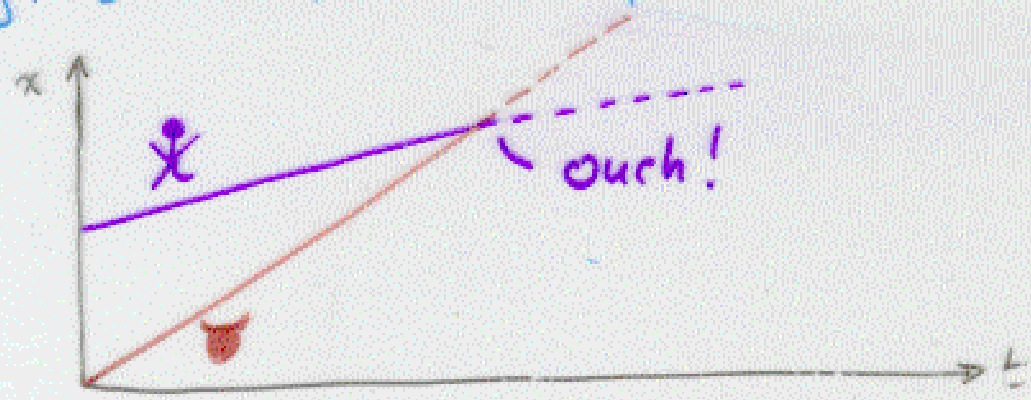
Total time $= t_2$

$\Rightarrow V_{av} = \dfrac{2(x_1 - x_0)}{t_2}$ in this case.

Uniform Speed ≡ Equal distances covered in equal time intervals (Galileo)

∴ Distance / time graph must be straight line

e.g. Bull chases man in field



Bull faster, but man has head start at $t=0$ ($x(0)>0$)

If speed is uniform

$$v = v_{av} = \frac{x(t) - x(0)}{t - 0} = \frac{x(t_1) - x(t_0)}{t_1 - t_0}$$

$$\Rightarrow \boxed{x(t) = x(0) + vt}$$

Equation of Uniform Motion

e.g. For man, if $v = 4\,m/s$, $x(0) = 18\,m$

+ for bull, if $v = 7\,m/s$, $x(0) = 0$

- when does the bull catch the man?
+ where?

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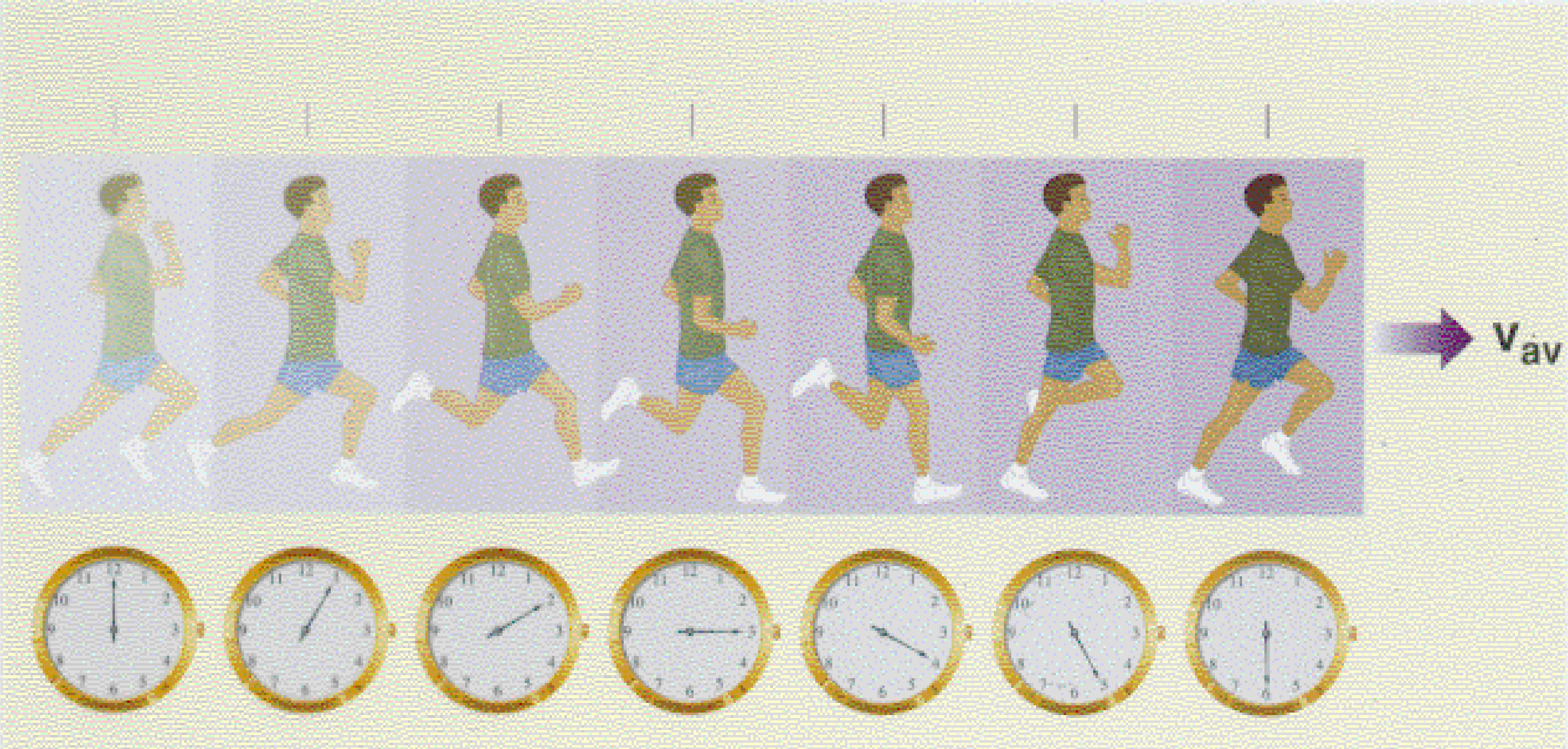
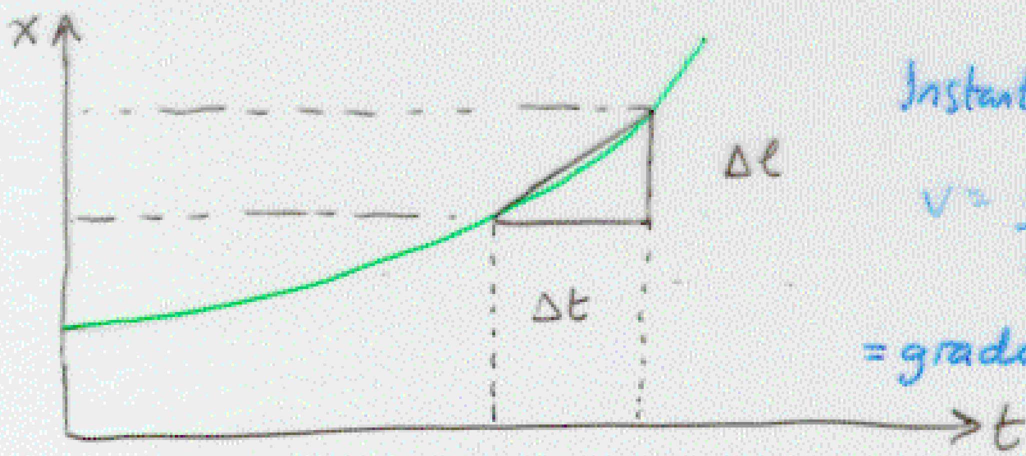
Figure 2.2

Runner at a constant speed



Instantaneous Speed ≡ rate of change of distance with time

e.g. car's speedometer measures distance Δl covered in short time Δt

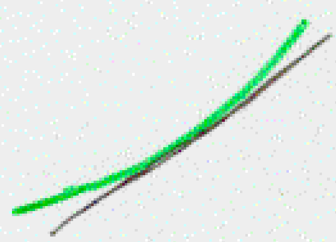


Instant. speed

$$v = \frac{\Delta l}{\Delta t}$$

= gradient of Δ.

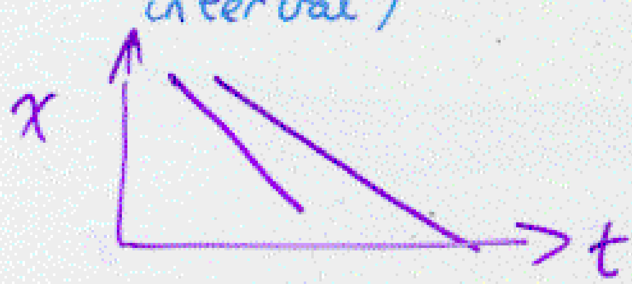
As interval $\Delta t \to 0$, $v = \dfrac{\Delta l}{\Delta t} \to \dfrac{dl}{dt}$



= gradient of tangent to curve

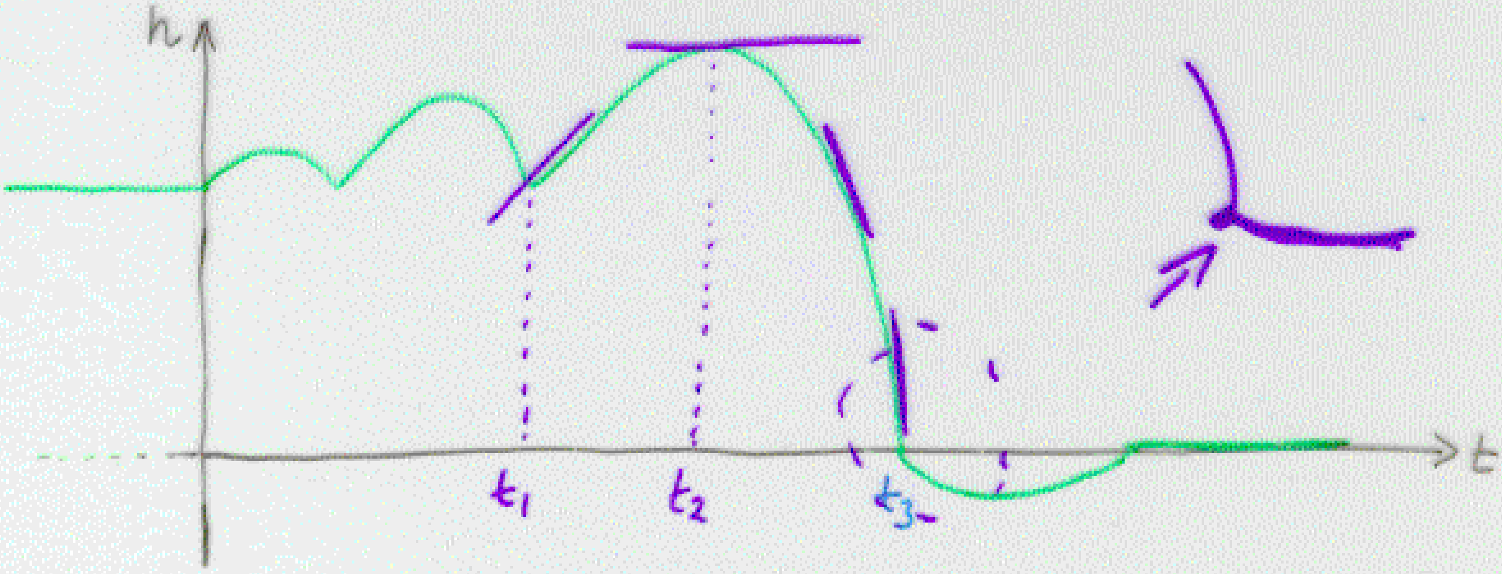
As gradient (slope) increases, v increases

(i.e. more distance covered in same time interval)



Analyzing distance - time graphs

e.g. A diver jumps off springboard into pool:



h : height above water surface

$t \leq 0$: diver still, $v = \frac{dh}{dt} = 0$ (flat)

$t = 0$: " jumps up suddenly, $v = \frac{dh}{dt} > 0$

$t = t_1$

$t = t_2$ diver begins to fall, $v = \frac{dh}{dt} = 0$ (for an instant)

$t > t_2$ diver falling, $v = \frac{dh}{dt} < 0$ (negative slope)

- slope steepens as diver falls faster (i.e. v becomes more negative)

$t \geqslant t_3$: splash! v still < 0, but suddenly reduced in magnitude as water slows diver