Gravitation cont/d.

Circular orbits: $F_c = F_G$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

(Choose $M \gg m$ so large mass M does not move)

$\Rightarrow$ Orbital speed $v = \sqrt{\frac{GM}{r}}$

(Measure) Orbital period $T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{GM}}\sqrt{r} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ (*)

So measure $T, r \Rightarrow$ value for M

e.g. Earth / Sun : $\left.\begin{array}{l} T = 1 \text{ year} \\ r = 149 \times 10^{9}\,\text{m} \end{array}\right\} \Rightarrow M_{sun} \approx 2 \times 10^{30}\,\text{kg}$

Periods of the other planets also obey (*)

– a good test that $F_G = \frac{GMm}{r^2}$ for

$r \sim$ few m (Cavendish expt.) out to

$r \sim 10^{12}$ m (outer planets)

24 19/2

Interesting orbits around the earth, $M_E = 6 \times 10^{24}$ kg

Using $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM_E}} \cong 3.14 \times 10^{-7} r^{3/2}$

and $R_E = 6400\text{km} = 6.4 \times 10^6\text{m}$

- Space Shuttle/ISS orbits at $h = 600$km

$$\Rightarrow r = R_E + h = 7 \times 10^6 \text{m}$$

So $T = 5815\text{s} \approx 97$ min.

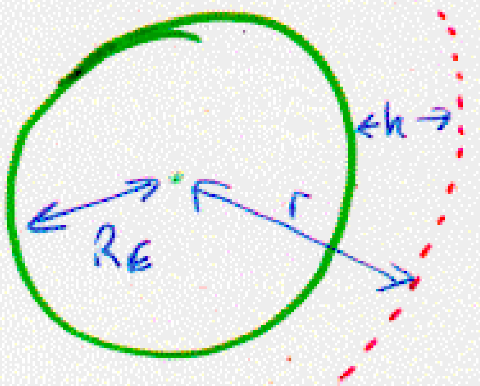
(and $v = \dfrac{2\pi r}{T} = 7.56$ km/s)

- Geostationary satellite (phone, TV) orbits with $T = 24h = 86400s$ to stay above same point on earth. So $r = 35.5 \times 10^6$m (35000km)

$h = 29\,000$ km.

- Moon has $r = 384\,000$ km (laser ranging)

$\Rightarrow T = 2.36 \times 10^6\text{s} \simeq 27.5$ day



F_G and Work : Escape Speed

$F_G = \frac{GMm}{r^2}$ extends out to $r \Rightarrow \infty$
("Gravity sucks").

So can one ever escape planet's gravity? Yes!

Launch craft with initial $KE = \frac{1}{2} m V_0^2$ from surface ($r = R_p$)

To escape to $r \to \infty$, KE must do work against F_G

$$\text{Work required} = \int_{R_p}^{\infty} F_G . dr = \int_{R_p}^{\infty} \frac{GMm}{r^2} . dr$$

$$\Rightarrow KE = \tfrac{1}{2} m V_0^2 \geqslant \left[-\frac{GMm}{r} \right]_{R_p}^{\infty} = \frac{GMm}{R_p}$$

$\therefore$ Min. escape speed $V_0 \geqslant \sqrt{\frac{2GM}{R_p}}$ (~11 km/s for earth)

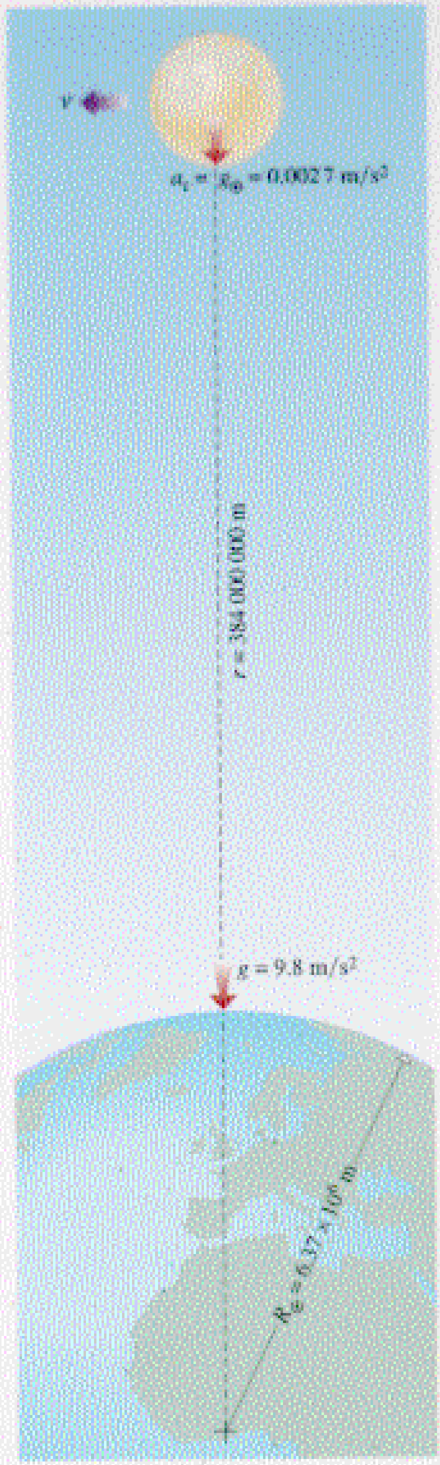
Note: On earth, since height $h << R_E$ we take

$F_G = \frac{GM_E m}{R_E^2} = mg$ as $\approx$ constant

$$\text{so work} = \int_{R_E}^{R_E+h} F_G . dr \approx F_G . h$$
$$= mgh$$

Figure 5.9

Acceleration due to gravity drops off inversely with distance squared



$= \frac{v^2}{r}$

Moon is "falling" towards earth even though $r \simeq$ constant.

Note: moon also "feels" F_G from Sun and orbits the Sun with $T = 1$ year.

Figure 5.10

Cavendish torsion pendulum for determining **G**