Week 6 Reading Quiz

1. An object moves along a circular path at constant speed. Its acceleration is:

a) Zero

b) Towards the center

c) Away from the center

d) Tangential to the path.

2. An athlete runs at 4 m/s and takes 300s to complete a circular path. What is the radius of this circle?

a) 1200 m.

b) $1200 \times \pi$ m.

c) $\frac{600}{\pi}$ m.

d) $600 \times \pi$ m.

$C = 1200\,m$
$= 2\pi r$
$r = \frac{1200}{2\pi}$

3. The moon has less mass than the earth. Relative to the gravitational force of the moon on the earth, the gravitational force of the earth on the moon is:

$$F = \frac{G m_1 m_2}{r^2}$$

a) Less.

b) More.

c) The same.

d) Zero.

4. Compared to the Earth, the outer planets orbiting the Sun:

✓ a) Have a lower orbital speed

✓ b) Feel a lower gravitational force from the Sun

✓ c) Have a longer orbital period

d) All three...

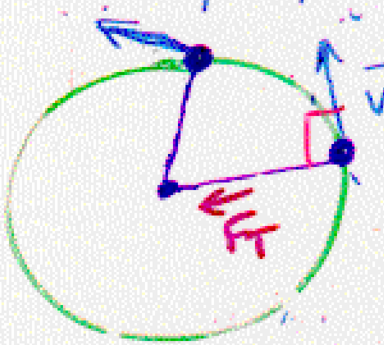
Curvilinear Motion (Ch 5)

So far: Newtonian Dynamics with
$\vec{a}$ either constant (or zero)
e.g. $\vec{a} = \vec{g}$; $\vec{a} = \frac{\vec{F} - \vec{F}_F}{m}$ etc.

Resulting path of object can still be a curve
(e.g. Parabolic trajectory for constant $\vec{g}$).

But what if $\vec{a}$, $\vec{F}$ change <u>direction</u> as well as magnitude?

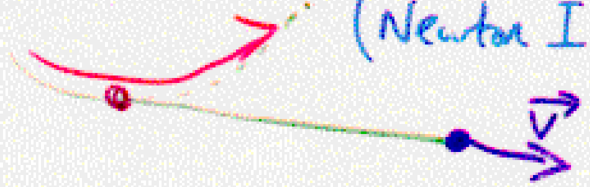
e.g. Whirl stone on string in circular path:



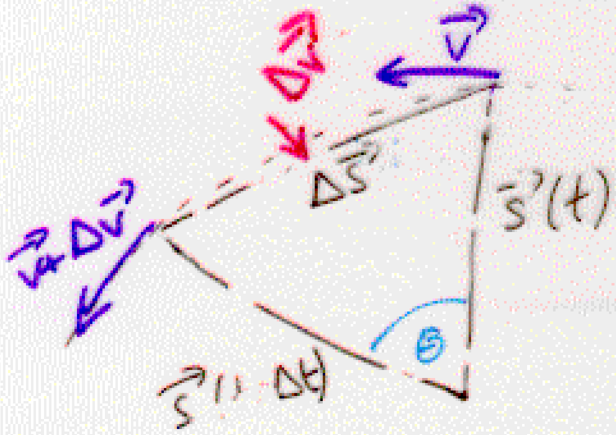
$\vec{v}$ tangential to path as before.
$|\vec{v}|$ constant but direction changes

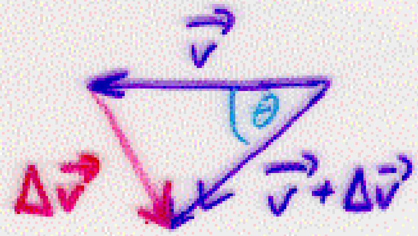
$\Rightarrow$ <u>there must be a force</u> acting to change $\vec{v}$.

<u>Tension $\vec{F}_T$</u> keeps motion in a circle.
Break string ($\vec{F}_T = 0$) $\Rightarrow$ (Newton I)



Circular Motion: Centripetal Acceleration





Analysis: In time Δt, object moves through angle θ and $\vec{v}$ changes direction by θ.

From similar Δs:
$$\frac{\Delta v}{v} = \frac{|\Delta\vec{s}|}{|\vec{s}|} = \frac{|\Delta\vec{s}|}{r}$$

since $|\vec{s}|$ = radius(r). Divide by Δt:

$$\Rightarrow \quad \frac{\Delta v}{\Delta t} = \frac{|\Delta\vec{s}|}{\Delta t}\,\frac{v}{r}$$

= speed v

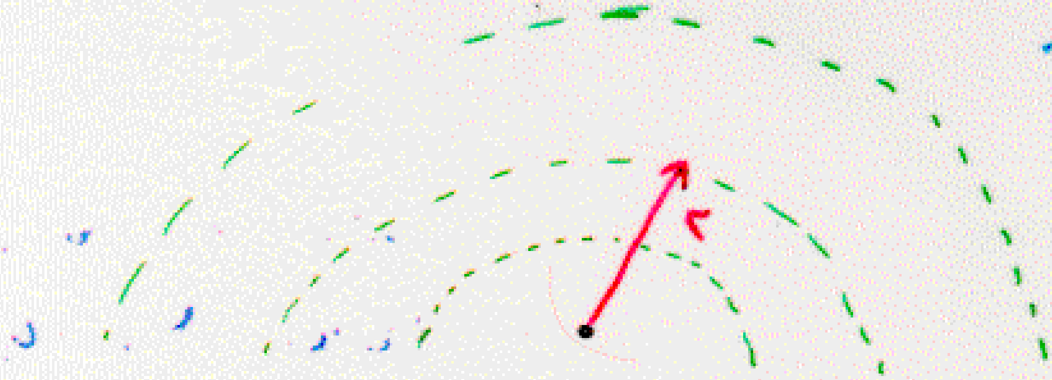
$$\boxed{\text{Centripetal Acceleration} \quad a_c = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}}$$

directed along $\Delta\vec{v}$, $\perp$ to path, i.e. towards center of circle

Centripetal Force

e.g. Race around circular track:

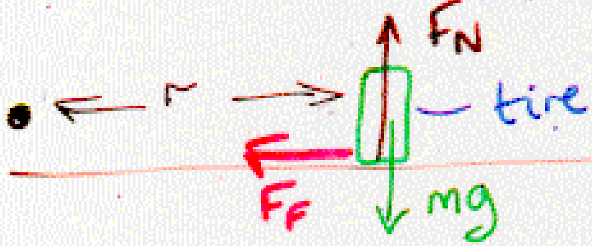
Top view.



Force required to provide $\vec{a}_c$ is $F_c = ma_c = \frac{mv^2}{r}$

Note: $F_c = 0$ when $v = 0$, $F_c \to 0$ as $r \to \infty$ ✓)

e.g. Car on flat race course enters curve



F_c is provided by friction force $F_F \leq (\mu F_N = \mu mg)$

i.e. $F_c = \frac{mv^2}{r} = F_F$

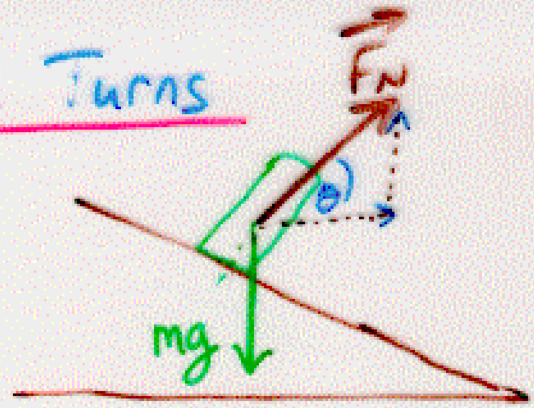
Tire will not skid as long as $F_c = F_F \leq \mu mg$

i.e. $\frac{\cancel{m}v^2}{r} \leq \mu \cancel{m} g$:

$\Rightarrow \quad v \leq \sqrt{\mu g r}$, max. safe speed of curve.

Banked Turns

Side view



Instead of friction, use component of $\vec{F}_N$ to supply the needed $F_c = \frac{mv^2}{r}$. (Doesn't work in space!)

Equate: Horizontal $F_c = \frac{mv^2}{r} = F_N \cos\theta$ (1)

Vertical (no motion) $F_V = mg \sin\theta$ (2)

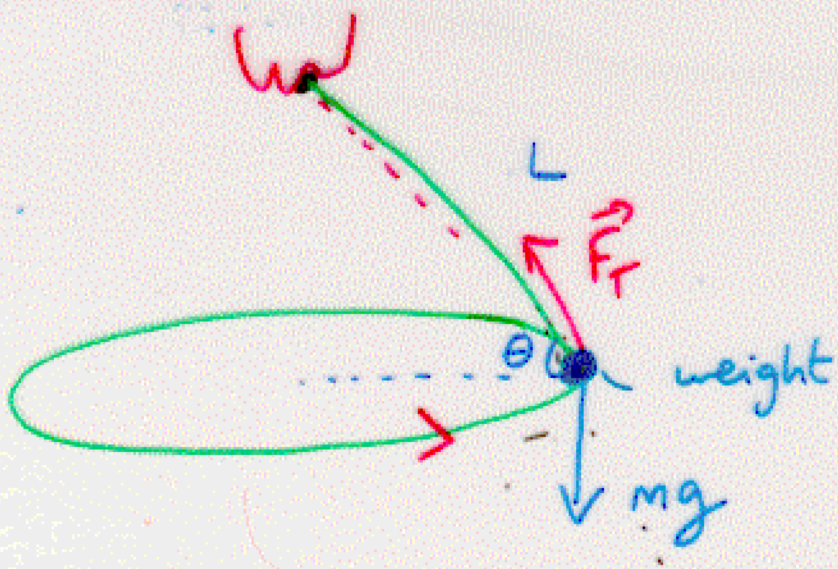
Divide (1) ÷ (2) $\Rightarrow$ $\tan\theta = \frac{gr}{v^2}$ — no friction required!

[In fact, $\tan\theta = \frac{dy}{dr} = \frac{gr}{v^2} \Rightarrow y = \frac{1}{2}\frac{gr^2}{v^2}$: parabola]

$\tan\theta = \frac{dy}{dr} = \frac{gr}{v^2}$

Velodrome track is banked for a given speed v

Example / Demo: Spinning Lariat



Tensile force F_T balances weight and provides F_c

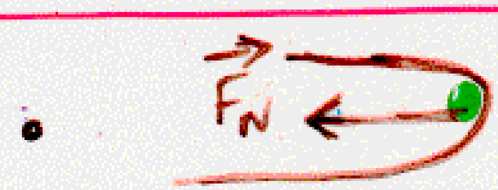
Vertical: $F_T \sin\theta = mg$

Horiz: $F_T \cos\theta = F_c = \frac{mv^2}{r}$

Divide $\Rightarrow$ angle $\tan\theta = \frac{gr}{v^2}$ c.f. banked track

As rope spins faster, $v\uparrow$ so $\theta\downarrow$.

c.f. Centrifuge with $\theta = 0$:



Force on sample $F_N = \frac{mv^2}{r} \gg mg$

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Vertical Rotations under Gravity

Bucket on string

Car on track

At top of loop the loop :

Net inward force

$F_c = \frac{mv^2}{r} = F_T + mg$: $F_c = \frac{mv^2}{r} = F_N + mg$

$\Rightarrow$ tension $F_T = \frac{mv^2}{r} - mg$: normal force $F_N = \frac{mv^2}{r} - mg$

For taut string $F_T \geqslant 0$: To stay on track $F_N \geqslant 0$

$$\Rightarrow \frac{mv^2}{r} \geqslant mg$$

or $$v^2 > gr$$

Otherwise (v too small), object starts to free-fall.

Figure 5.5

A banked road