Impulse changes Momentum ; Work changes K.E.

$\boxed{\int F.dt = \Delta mv}$ $\boxed{\int F.dx = \Delta(\frac{1}{2}mv^2)}$

e.g. roller coaster with $m = 600\text{kg}$, $v = 22.6\text{m/s}$ brought to rest by constant $F = 5600\text{N}$ [quiz 3]

Force F must act over time to change momentum (Newton II)

$$F\Delta t = mv - 0 = \Delta p \quad (1)$$

Same force F acts over distance Δx to do work against K.E.

$$F\Delta x = \frac{1}{2}mv^2 - 0 \quad (2)$$

(Can see that if force F ↑, both Δt and Δx ↓)

But can same force F satisfy both eqn. (1) and (2)?

Yes! From (1) and (2) : $\Delta x = \frac{1}{2}\underbrace{\left(\frac{F}{m}\right)}_{\text{accel.!}}\Delta t^2$

– which is what we expect for constant force / accel.

For values above, we find $a = \frac{F}{m} = 9.33\text{ m/s}^2$

$\Delta t = 2.42\text{s}$, $\Delta x = 27.3\text{m}$ and

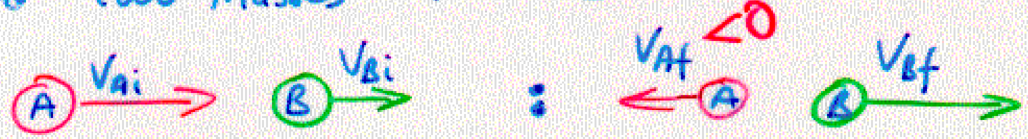
$\Delta KE = 150.5\text{ kJ}$ converted to heat in brakes

15/12 [illegible]

Collisions and Conservation of Momentum

- 'Collision' $\equiv$ any interaction where momentum transfered.
- Total momentum (before, during, after) = constant if no external forces, i.e. $\Delta \vec{P} = 0$.

e.g for two masses A and B



Conservation $\Rightarrow$ $\underline{M_A V_{Ai} + M_B V_{Bi} = M_A V_{Af} + M_B V_{Bf}}$

Note: $\vec{p} = m\vec{v}$ is a vector, so use correct signs for V_A, V_B

However: 1 equation, 2 unknowns (V_{Af}, V_{Bf})

So need to specify type of collision to determine motions completely:

Inelastic "splat" collisions: $(KE)_f < (KE)_i$: energy lost to heat, deformation

Elastic collisions: $(KE)_f = (KE)_i$: kinetic energy conserved

Example: McGwire's 70th Home Run (p. 284)

Before:

Ball ($m_A = 0{\cdot}4\,kg$) pitched at $V_{Ai} = -40{\cdot}25\,m/s$

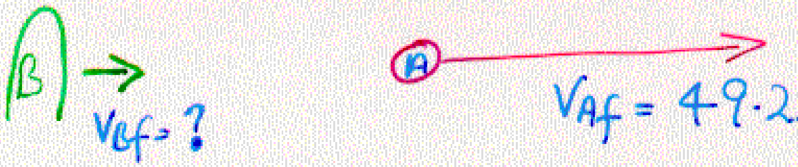
Bat ($m_B = 1\,kg$) swung at $V_{Bi} = +38\,m/s$

$\Rightarrow$ initial momenta $\quad P_{Ai} = m_A V_{Ai} = 0{\cdot}4 \times (-40{\cdot}25) = -16{\cdot}1\,kg\,m/s$

$P_{Bi} = m_B V_{Bi} = 1{\cdot}0 \times 38 \quad = 38$ "

$\therefore$ Total Momentum $P_i = P_A + P_B \quad = \underline{\underline{+21{\cdot}9\,kg\,m/s}}$

After:



Use $P_f = P_i = 21{\cdot}9\,kg\,m/s$ where $P_f = m_A V_{Af} + m_B V_{Bf}$

Given for ball: $P_{Af} = m_A V_{Af} = 0{\cdot}4 \times 49{\cdot}25 = +19{\cdot}7\,kg\,m/s$

$\Rightarrow$ bat: $P_{Bf} = 21{\cdot}9 - 19{\cdot}7 = 2{\cdot}2\,kg\,m/s \Rightarrow \underline{V_{Bf} = 2{\cdot}2\,m/s}$

Can now find <u>impulse</u> on ball (= −impulse on bat)

e.g. $\Delta P_A = P_{Af} - P_{Ai} \quad = 35{\cdot}6\,kg\,m/s$ or $\underline{35{\cdot}6\,Ns}$

$= \int F\,dt$ or $F_{av}\,.\,\Delta t$

So collision time $\Delta t = 1\,ms \Rightarrow F_{av} = \dfrac{35{\cdot}6\,Ns}{10^{-3}\,s} = 35{\cdot}6\,kN!$

(shattering!)

Can also find $(KE)_f$ vs. $(KE)_i$ .....

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Elastic Collisions

In 1-D: billiard balls, tennis racket + ball etc.

(Let B be stationary at first, $v_{Bi} = 0$)

$$\Delta \vec{P} = 0 \text{ (momentum)}: \quad m_A v_{Ai} + 0 = m_A \underline{v_{Af}} + m_B \underline{v_{Bf}}$$

$$\Delta KE = 0 \text{ (kinetic energy)}: \quad \tfrac{1}{2} m_A v_{Ai}^2 = \tfrac{1}{2} m_A v_{Af}^2 + \tfrac{1}{2} m_B v_{Bf}^2$$

Can solve for final velocities v_{Af}, v_{Bf}.

We find (eq. 7.9, 7.10, 7.11)

$$v_{Af} = \frac{(m_A - m_B)}{(m_A + m_B)} v_{Ai} \qquad v_{Bf} = \frac{2 m_A}{(m_A + m_B)} v_{Ai}$$

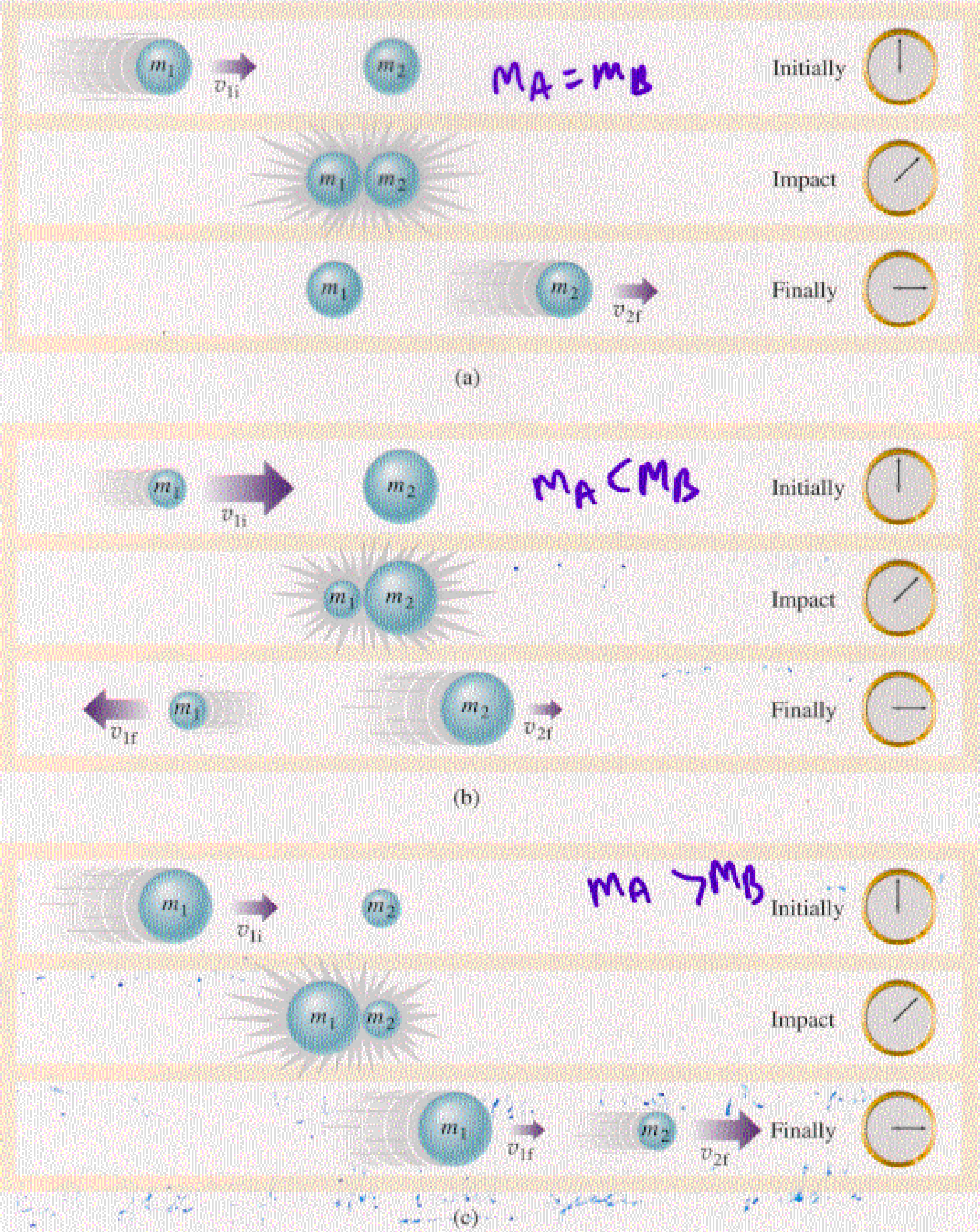
Note: $v_{Bf} > 0$ always (B knocked forwards), but

v_{Af} can be < 0 if $m_A < m_B$ (rebounds backwards)

Also find $v_{Bf} - v_{Af} = v_{Ai}$: "relative speeds" are the same before/after collision.

Figure 7.13

Three elastic collisions



Elastic Collision Features

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1) $m_A = m_B$: Then $v_{Af} = 0$, $v_{Bf} = v_{Ai}$

i.e. all momentum (+ k.e.) transfered A → B, A is "stopped"

– most efficient for k.e. transfer

(e.g. club-head / golf ball)

2) $m_A < m_B$: Then $v_{Af} < 0$ (rebounds backwards)

and momentum transfer to B $m_B v_{Bf} = m_A (v_{Ai} - v_{Af})$

Note: if $m_A << m_B$, $v_{Af} = -v_{Ai}$ – perfect rebound

(e.g. throw ball at a ship, hit bowling ball with golf club!).

3) $m_A > m_B$: Then $v_{Af} > 0$ ("follows through"), $v_{Bf} > v_{Af}$

– lighter mass flies off

For $m_A >> m_B$, $v_{Bf} \approx 2 v_{Ai}$ (e.g. ping-pong

ball flies off with twice speed of paddle).