Potential Energy: Work Stored by Position

Throw a ball horizontally with speed v

OR

In each case ball does work $W = \frac{1}{2} m v^2$

(against gravity, or against spring)

⇒ K.E. stored as Potential Energy

Ball rolls downhill:

Gravity does work $= F_W \cdot h$

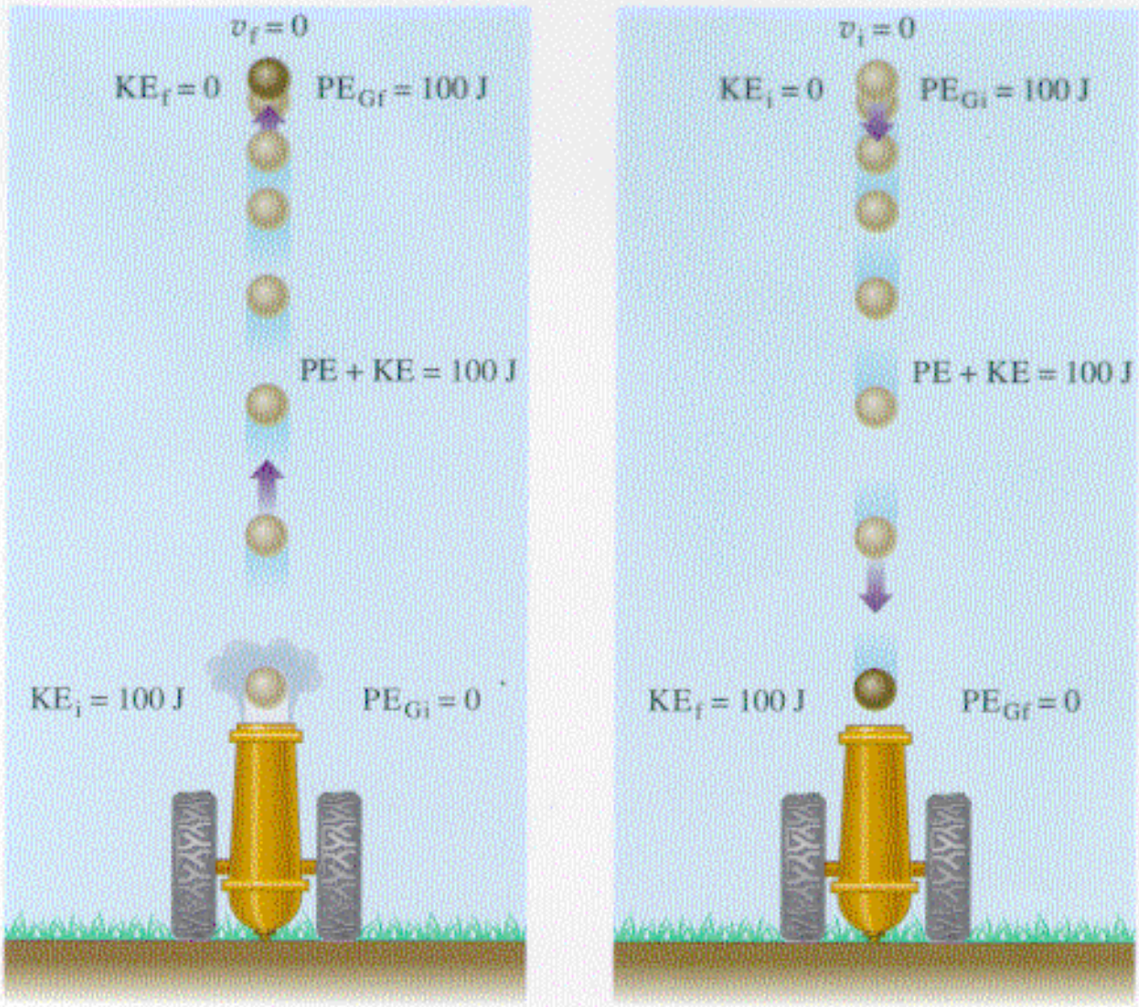
$= mgh$ on ball

OR compressed spring expands, doing work $F_{sp} \cdot \Delta x$

potential energy stored by virtue of position (or "configuration")

Figure 6.21

KE and PE_G for a cannonball fired upward



Problem Solving using Energy

e.g. Dodder and ramp : $mg = 1000\,N$, $\theta = 15°$, $l = 8m$

a) No friction : Find force F

To move container a distance l along ramp:

Work $W = F.l$ = change in P.E. $= mgh$

$\therefore F = \frac{mgh}{l} = mg \sin\theta$ as before

b) Add friction $F_F = \mu mg \cos\theta$ against motion

Now work required $F.l = mgh + F_F.l + \frac{1}{2}mv^2$

$\Rightarrow F = mg \sin\theta + F_F$ as before

c) box slides down ramp :

> K.E. at bottom = loss of P.E. − work done against friction

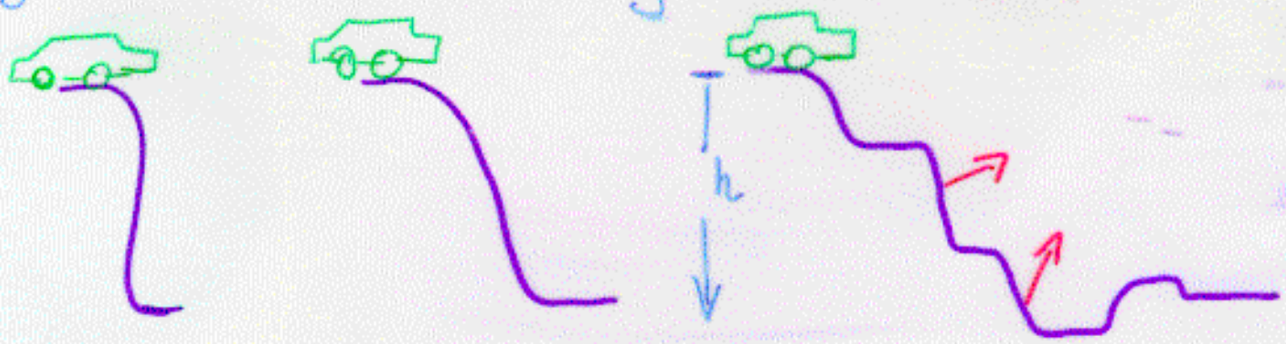
$\Rightarrow \frac{1}{2}mv^2 = mgh - F_F l$

$\Rightarrow v^2 = 2gl(\sin\theta - \mu\cos\theta)$, as before

$\tan\theta = \mu$

Speed, K.E. and P.E.

e.g. A car can descend 5 m by 3 methods.



If no friction, speed at bottom given by

ΔKE $\frac{1}{2} m (v^2 - v_0^2) = mgh$, i.e. the same!

or $v^2 = v_0^2 + 2gh = 0 + 2 \times 10 \times 5$

$\Rightarrow v = 10 m/s$, depends only on h.

(But time taken increases with increasing slope)

Careful! What if we give car initial "push" $v_0 = 3 m/s$

– is speed at bottom then $10 + 3 = 13 m/s$???

No: Speed $\neq$ energy

$v^2 = v_0^2 + 2gh = 3^2 + 2 \times 10 \times 5$

$\Rightarrow v = \sqrt{109} = 10.44 m/s$, not $13 m/s$.

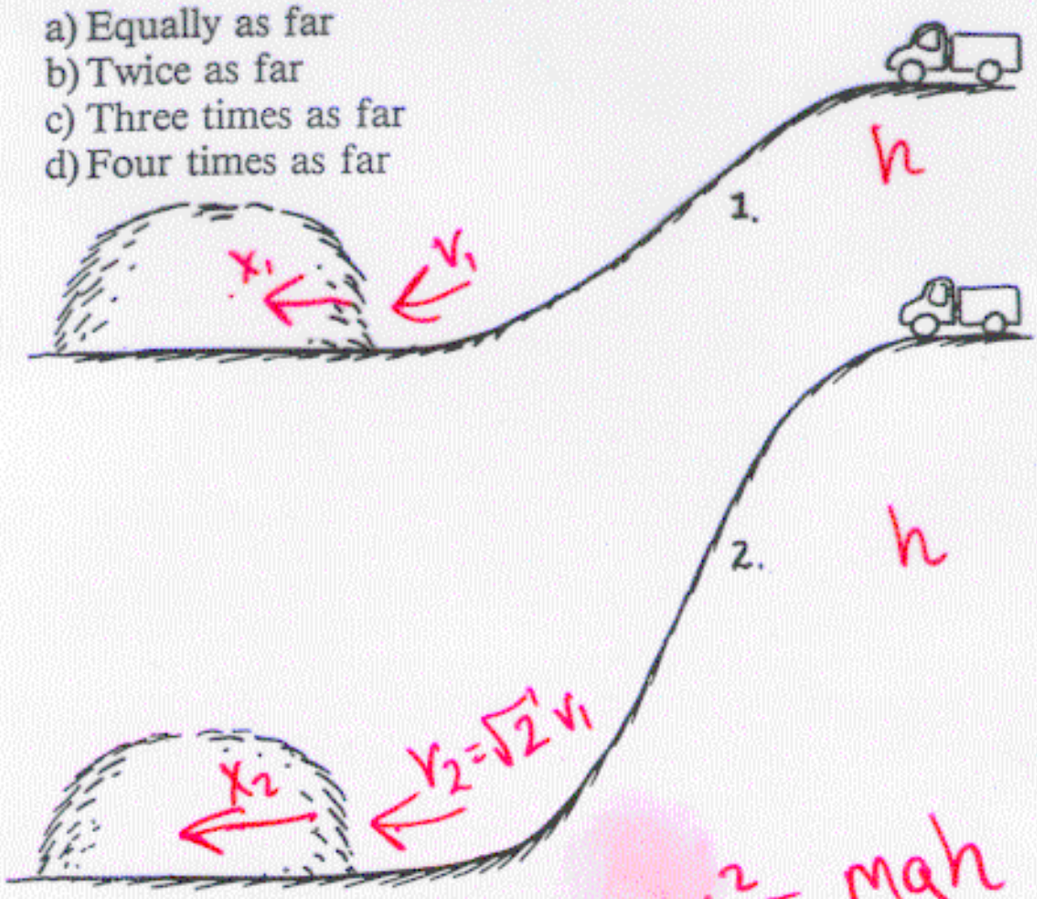
Add friction? $\frac{1}{2} m (v^2 - v_0^2) = mgh - \int F_F \cdot dl$

$\Rightarrow$ reduces v at the bottom.

PENETRATION

A truck initially at rest rolls down Hill 1 into a very big haystack. Another identical truck also rolls from rest down Hill 2, twice as high, into an identical haystack. Compared to the truck on Hill 1, how much farther does the truck on Hill 2 penetrate into the stack?

a) Equally as far
b) Twice as far
c) Three times as far
d) Four times as far



$W_H = F_H x = \frac{1}{2} m v^2 = mgh$

$x_2 = 2x_1$

Power = Rate of Doing Work

DEFINE: **Power** $P = \dfrac{\text{Work done}}{\text{time taken}}$

Units J/s
(= N m/s)
= Watts (W)

e.g. If one engine raises a 10kg water bucket up a well with $h = 20m$ in 30s.

Work $W = mgh = 10kg \times 10m/s^2 \times 20m = 2000\,J$

Power $P = \dfrac{W}{\Delta t} = \dfrac{2000\,J}{30s} = 66.7\,W$.

Another motor does the same task in ½ the time (15s)

⇒ Power is double = 133.4 W

~ ⅓ hp

c.f. garage door opener, or healthy adult.

Power measures rate of transfer of work-energy from force provider to point of application