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Friction : Blessing and a Curse !

Friction between surface and wheels (or feet) required for motion !

Wheel / foot exerts force backwards on ground $\leq \underline{\underline{\mu_s F_N}}$ (no slipping) : (ground moves backwards e.g. tread mill)

$\Rightarrow$ equal / opposite force of ground on wheel or foot !

On icy surface, $\mu \approx 0 \Rightarrow$ no motion possible.

Horse + cart :

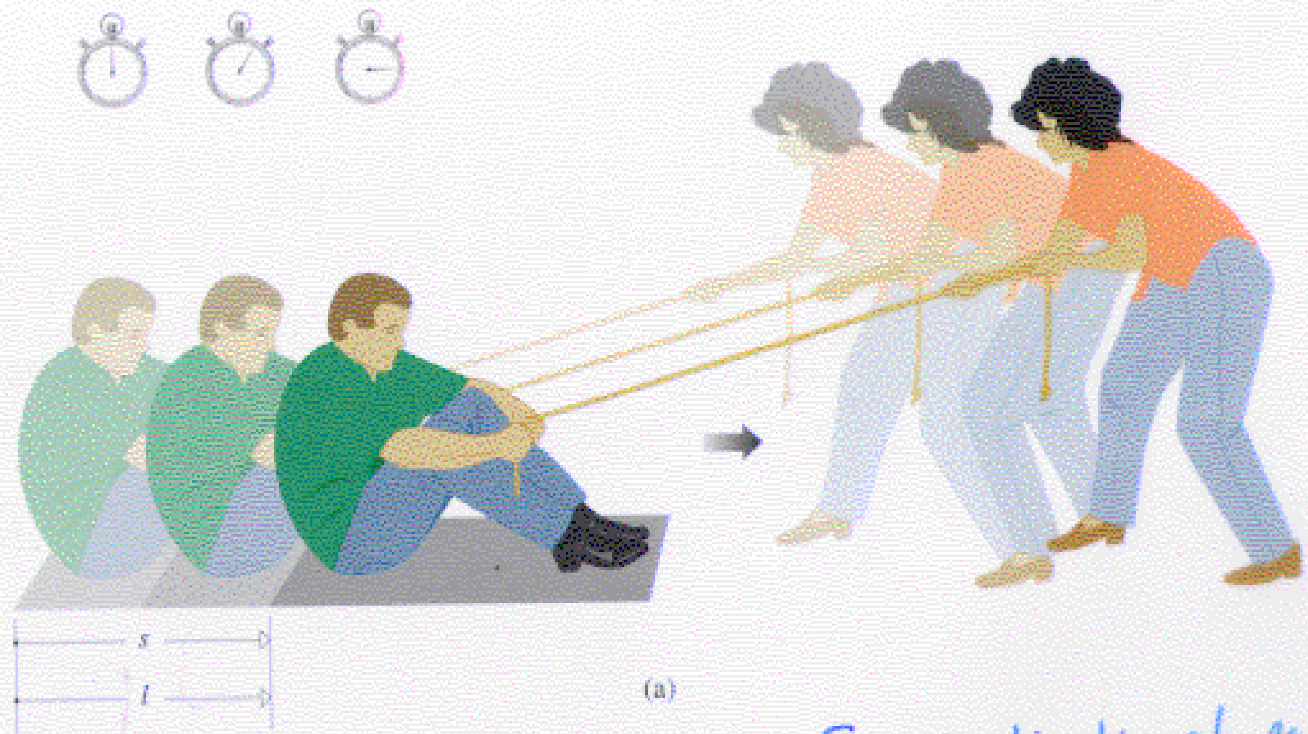
$\xrightarrow{F_T}$ $\quad \xleftarrow{} \vec{F_T} = -\vec{F}_{FH}$

Friction force at hoofs > friction force at wheels

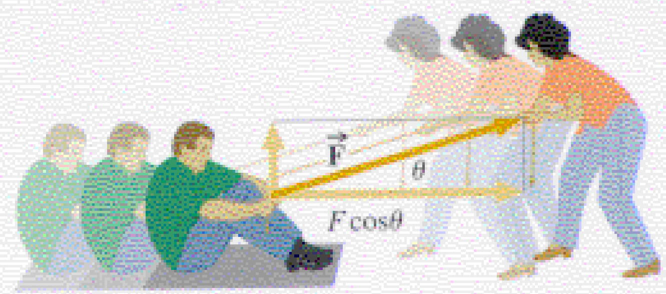
$\Rightarrow$ net force on cart $\Rightarrow$ cart begins to move.

$a = F / M_c$

Figure 6.3
Force is a vector with two perpendicular components



(a)



(b)

Force in direction of path l

$= F\cos\theta$

$\Rightarrow W = Fl\cos\theta$

Work and Energy (Ch. 6)

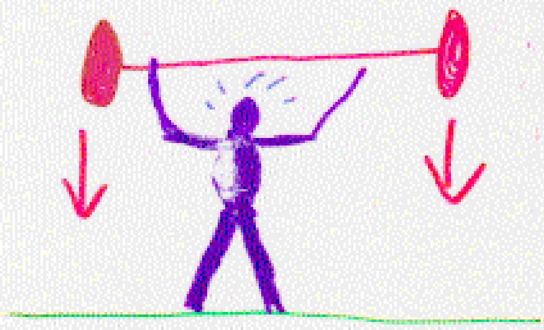
Vaguely: Energy is a measure of change to a system. (More energy transferred in/out $\Rightarrow$ greater change)

Work: Transfer of energy when a force moves its point of application

DEFINE: Work done W = Force × path length moved

$W = F \cdot l$ units: Nm or Joules (J)

Note: No motion $\Rightarrow$ no work!



Upward force = mg
but work = 0

$F = mg$ $W = mgh$

Applications of Work: Gravity

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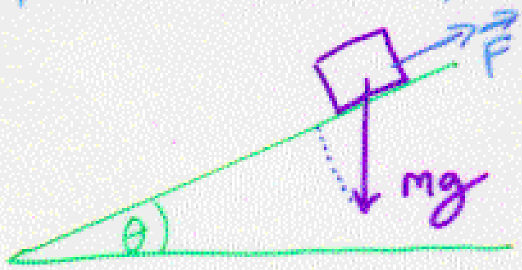
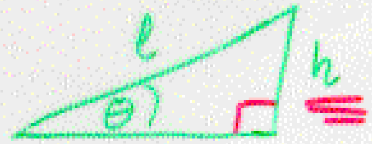
e.g. Docker pushes container up a ramp (quiz 2)





(No friction for now) Force required $F = mg\sin\theta$

If ramp has length ℓ, Work done $W = F\ell\sin\theta$

But $\ell\sin\theta$ = height of ramp h

$\Rightarrow W = F.h = mgh$, independent of θ

OR just consider motion in direction of force $\vec{mg}$ (i.e. vertical direction only). For a cyclist:

(No friction). At any point, work done by cyclist

$W = mgh$ ($mg\Delta h$)

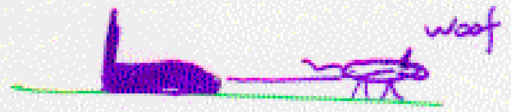
$\therefore$ as height h increases, cyclist does positive work

" " decreases, " " negative "

Note: Work measured relative to initial state ($h=0$)

Applications of Work: Friction

e.g. Drag sled across ice at constant speed



$F_F = \mu_K F_N$

- Friction force $F_F = \mu_K F_N$ opposes applied force from dog
- Work is done on sled and on ground (both heat up)
- Net work done by dog $W = F_F \cdot \ell$

Note: path length (= odometer reading) ℓ always increases, so work done <u>does</u> depend on path. (>0)

e.g.

Top view

For $\ell = 10\,km$ round trip with $\mu = 0.1$, $mg = 500\,N$

Dog's work

$$W = F_F \cdot \ell = \mu m g \ell$$

$$= 0.1 \times 500 \times 10^4\,m = 5 \times 10^5\,J$$

1 Calorie $\equiv 4.2\,kJ$, so dog expends $\frac{5 \times 10^5}{4.2 \times 10^3}$

$= 119$ Calories

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Work as a Vector Product

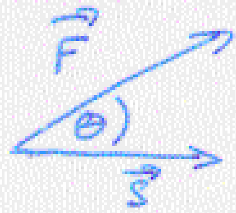
Pulling suitcase:

Drag case along with force F a distance ℓ

($F > \mu_k F_N \Rightarrow$ case will accelerate).

Work done $W = F\ell \cos\theta$

In general for a vector displacement $\vec{s}$



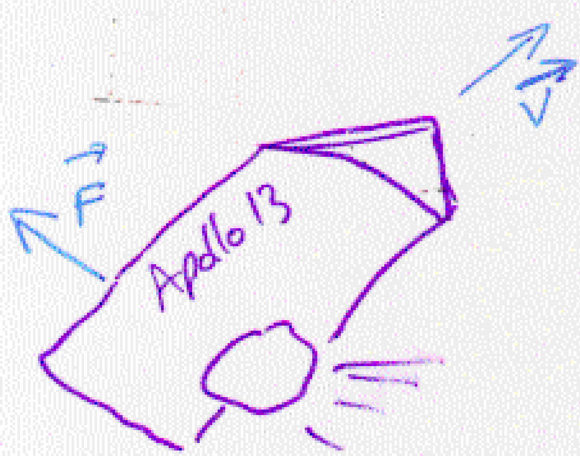
The VECTOR "DOT" PRODUCT defined as

$$\vec{F}\cdot\vec{s} = Fs\cos\theta$$

So work $W = \vec{F}\cdot\vec{s}$

Note: Component of $\vec{F}$ $\perp$ $\vec{s}$ does no work (no motion)

e.g.



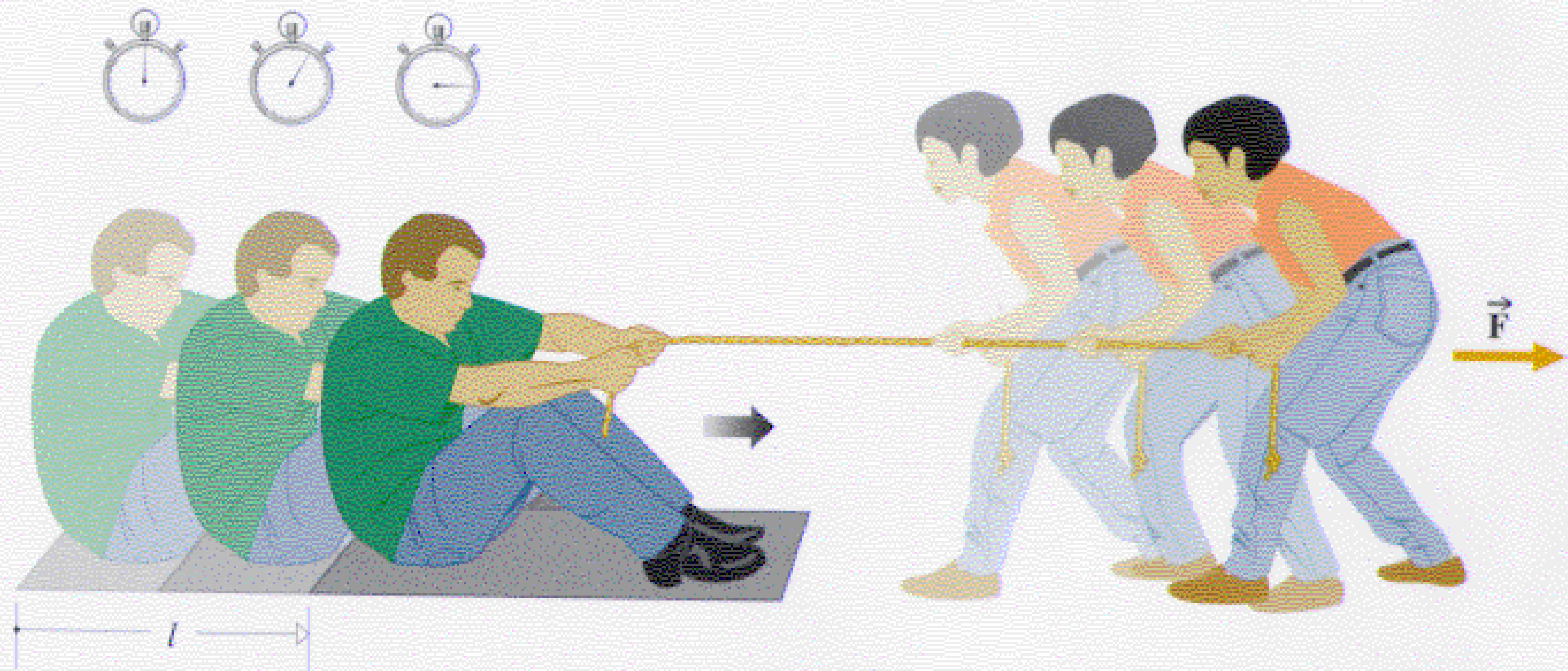
Here $\vec{F} \perp \vec{v}$

$\theta = 90^\circ$ so

$\vec{F}\cdot\vec{s} = Fs\cos 90^\circ = 0.$

Figure 6.1

Work done by a force $\vec{F}$



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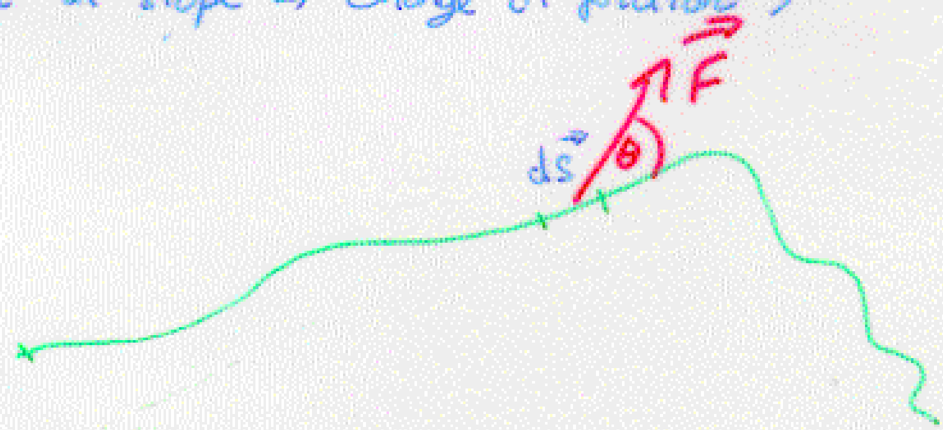
Work as an Integral

If Force $\vec{F}$ changes along path (e.g. due to change in slope $\Rightarrow$ change in friction)



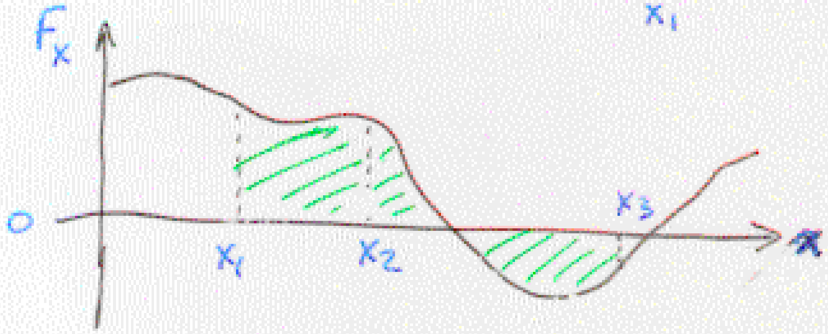
Work done over small displacement $d\vec{s}$ is

$$dW = \vec{F} \cdot d\vec{s}$$

$\therefore$ Integrating along path $W = \int \vec{F} \cdot d\vec{s}$

e.g. Motion in 1-dimension $|d\vec{s}| = dx$,

$$\Rightarrow \text{Work } W_{12} = \int_{x_1}^{x_2} F_x \cdot dx$$



also $W_{13} = W_{12} + W_{23}$ etc.

i.e. additive

Power as a time-derivative

Power P = rate of Work $\quad P = \dfrac{dW}{dt}$

If body moves $d\vec{s}$ in time dt

Work done $dW = \vec{F}.d\vec{s}$ as before

$$\therefore \text{Power} \quad \boxed{P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v}}$$
$$= Fv\cos\theta$$

e.g. to move a 15 kg vacuum cleaner over a carpet at 2 m/s with $\mu = 0.4$:

$$P = Fv = \overbrace{\mu m g}^{F_F} v = 0.4 \times 15 \times 10 \times 2$$
$$= 120\,W$$

e.g. If vehicle experiences constant drag force F_F

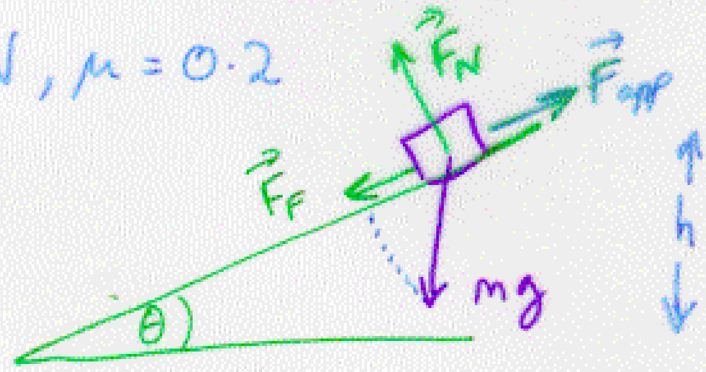
Power required to keep speed constant at v

is $P \geq F_F v$, so constant-power engine has a maximum speed $v \leq P/F_F$.

Example:

Docker pushes container up 15° slope for $l = 8\,m$

$mg = 1000\,N$, $\mu = 0.2$



Min. Work required $= F_{app} . l = \underbrace{mg\,h}_{W_{grav}} + \underbrace{F_F\,l}_{W_{friction}}$

$\therefore$ min. force $F_{app} = mg \frac{h}{l} + F_F \; (= \mu F_N)$

$= mg \sin\theta + \mu mg \cos\theta$

$= 258.8 N + 193.1 N$

$F_{app} = 452 N$

and work $= F_{app} \times 8m = 3616\,J$ or 0.86 Calories

At top, if container rolls down

Net force $= mg \sin\theta - F_F = mg(\sin\theta - \mu \cos\theta)$ $(\mu = 0)$

$\Rightarrow$ accel $a = g(\sin\theta - \mu\cos\theta) = 0.656\,m/s^2$ $(2.59\,m/s^2)$

$\therefore$ final speed $v^2 = v_0^2 + 2a.l \Rightarrow v = 3.24\,m/s$ $(6.44\,m/s)$

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Work and Kinetic Energy

To change speed of rigid body with a force requires work. Release force $\Rightarrow$ object continues (Newton I)

BUT object can now do work, e.g. by slowing down

e.g. Play catch with ball

Work is "stored" in ball's motion as Kinetic Energy, and released when brought to rest by braking force.

If Force $\vec{F}$ is constant and acts over distance x:

$W = F.x = \underbrace{max}_{\text{Newton II}}$: can relate this to speed of ball v

Since $v^2 = v_0^2(0) + 2ax = 2\frac{F}{m}x$ (Newton II)

$\Rightarrow W = max = \frac{1}{2}mv^2$

$\therefore$ $\boxed{\text{Kinetic Energy} \quad KE = \frac{1}{2}mv^2}$

In general Work = change in K.E. $W = \Delta KE$

Example: Work (energy) to accelerate a 500 kg car

(i) from $\overset{V_0}{0}$ to $\overset{V_1}{25}$ m/s (~0 to 60 mph)

(ii) from 25 to $\overset{V_2}{50}$ m/s

with no friction

(i) $W = \Delta KE = \frac{1}{2} m (V_1^2 - V_0^2) = \frac{1}{2}(500) \times 25^2$

average force × dist $F_{av} \cdot l_{01}$ $= 156\,kJ$

(ii) $W = \frac{1}{2} m (V_2^2 - V_1^2) = \frac{1}{2} 500 (50^2 - 25^2)$

$= 468\,kJ$

i.e. 3x the work (so 3x the distance for same F_{av})

even though time $t_2 - t_1 = \frac{V_2 - V_1}{a}$ is the same

i.e. $\Delta t = \frac{m \Delta V}{F}$

(Note: rockets can provide ~constant force F; most other engines → constant power $= \frac{dW}{dt}$)