PHYSICS 1a: HOMEWORK SOLUTIONS CHAPTER 8

8.1

Since $180° = \pi\,\text{rad}$,

$$1.000\,0° = (1.000\,0°)\left(\frac{\pi\,\text{rad}}{180°}\right) = 0.017\,453\,\text{rad}\,.$$

8.9

Use Eq. (8.1), $\theta = \ell/r$, to find the diameter ℓ of the circular area that the laser beam will illuminate on the surface of the Moon. Here $r = r_{\text{☾}} = 3.8 \times 10^8\,\text{m}$ and $\theta = 8 \times 10^{-4}\,\text{rad}$; so

$$\ell = r_{\text{☾}}\theta = (3.8 \times 10^8\,\text{m})(8 \times 10^{-4}\,\text{rad}) = 3 \times 10^5\,\text{m}\,.$$

This is about 30 km, or 20 mi.

8.11

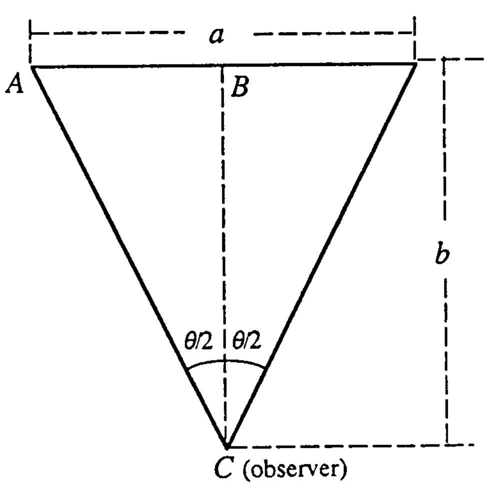
The standard TV picture has 525 horizontal scan lines so the separation between two adjacent lines is $\ell = 30\,\text{cm}/525 = 5.71 \times 10^{-4}\,\text{m}$ for a screen 30 cm high. The angle θ that any pair of adjacent scan lines subtend where you are is then $\theta = \ell/r$, where r is the distance between you and the TV screen. If you can no longer distinguish such two adjacent lines, then θ must be no more than $\theta_{\text{min}} = (1.0\,\text{min})(1°/60\,\text{min})(\pi\,\text{rad}/180°) = 2.91 \times 10^{-4}\,\text{rad}$, which is the smallest angle your eyes can resolve. Let $\theta_{\text{min}} = \ell/r_{\text{min}} \le \theta = \ell/r$, we may find r_{min}, the minimum distance you should sit in front of the TV:

$$r_{\text{min}} = \frac{\ell}{\theta_{\text{min}}} = \frac{5.71 \times 10^{-4}\,\text{m}}{2.91 \times 10^{-4}\,\text{rad}} = 2.0\,\text{m}\,.$$

8.15

In the figure shown to the right, the width of the car is denoted as a while the distance between the observer and the car is b. If the angle subtended by the car is θ as viewed by the observer, then in the right-angled triangle $\triangle ABC$

$$\tan\frac{\theta}{2} = \frac{\overline{AB}}{\overline{BC}} = \frac{a/2}{b}\,.$$



For $a = 2.00\,\text{m}$ and $b = 1.00\,\text{m}$, this gives $\tan(\theta/2) = (2.00\,\text{m}/2)/1.00\,\text{m} = 1.00$, so $\theta/2 = 45.0°$ and $\theta = 90.0° = 1.57\,\text{rad}$. If b is increases to 100 m, then $\tan(\theta/2) = (2.00\,\text{m}/2)/100\,\text{m} = 0.0100$, so $\theta/2 = 0.573°$ and $\theta = 1.15° = 0.020\,0\,\text{rad}$.

If you use the approximation suggested in the problem statement then from 100 m away $\theta \approx 2.00\,\text{m}/100\,\text{m} = 0.020\,0\,\text{rad} = 1.15°$, an excellent approximation; and from 1.00 m away $\theta \approx 2.00\,\text{m}/1.00\,\text{m} = 2.00\,\text{rad} = 115°$, a bad approximation.

8.21

Use Eq. (8.10): $v = r\omega$, where $r = r_{\oplus\odot} = 1.495 \times 10^{11}$ m and $\omega = 1.99 \times 10^{-7}$ rad/s (see the previous problem). Thus the linear speed of the orbital motion of the Earth is

$$v = r_{\oplus\odot}\omega = (1.495 \times 10^{11}\,\text{m})(1.99 \times 10^{-7}\,\text{rad/s}) = 2.98 \times 10^{4}\,\text{m/s}.$$

This is about 30 km/s, or 66×10^3 mi/h.

8.37

The angular speed ω of the train moving in a circle of radius r is related to its linear speed v via Eq. (8.10): $v = r\omega$. Solve for ω:

$$\omega = \frac{v}{r} = \frac{8.9\,\text{m/s}}{304.8\,\text{m}} = 0.029\,\text{rad/s}.$$

The centripetal acceleration of the train is

$$a_c = \frac{v^2}{r} = r\omega^2 = (304.8\,\text{m})(0.029\,2\,\text{rad/s})^2 = 0.26\,\text{m/s}^2.$$

8.43

The linear speed at the perimeter of each gear is identical to that of the other gears: $\omega_1 R_1 = \omega_2 R_2 = \cdots = \omega_5 R_5$. Also, the radius of a particular gear is proportional to the number of teeth it has. So for the first, fourth and fifth gears $(1500\,\text{rpm})(20) = \omega_4(25) = \omega_5(75)$, which gives $\omega_4 = 1200$ rpm and $\omega_5 = 400$ rpm.

In general, Let the number of teeth on the n-th gear be N_n, then from $\omega_1 R_1 = \omega_n R_n$ and $R_1/R_n = N_1/N_n$ we get $\omega_n/\omega_1 = R_1/R_n = N_1/N_n$. Solve for ω_n:

$$\omega_n = \left(\frac{N_1}{N_n}\right)\omega_1.$$

If n is an odd number, then the n-th gear rotates in the same sense as the first one. Otherwise it rotates in the opposite sense.

8.70

Use Eq. (8.26) to find the torque: $\tau_0 = Fr_\perp$. Here $F = 20$ N. For the elbow (E) $r_\perp = 23\,\text{cm} + 6\,\text{cm} = 29$ cm, so

$$\tau_E = Fr_\perp = (20\,\text{N})(0.29\,\text{m}) = 5.8\,\text{N}\cdot\text{m},$$

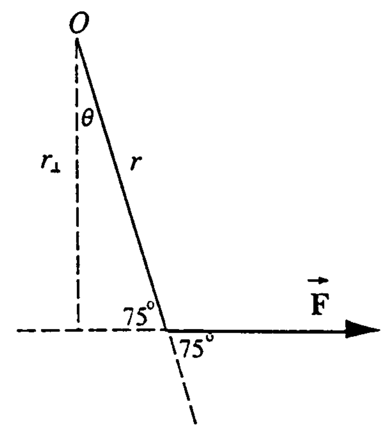
clockwise; and for the shoulder (S) $r_\perp = 23\,\text{cm} + 6\,\text{cm} + 28\,\text{cm} = 57$ cm, so

$$\tau_S = Fr_\perp = (20\,\text{N})(0.57\,\text{m}) = 11\,\text{N}\cdot\text{m},$$

also clockwise.

8.72

A simplified version of Fig. P72 is shown to the right, with $F = 80\,\text{N}$, $r = 0.06\,\text{m}$, and $\theta = 90° - 75° = 15°$. The lever arm for the torque τ_0 about point O due to the force F is $r_\perp = r\cos\theta$; so



$$\begin{aligned}\tau_0 &= Fr_\perp = Fr\cos\theta \\ &= (80\,\text{N})(0.06\,\text{m})(\cos 15°) \\ &= 4.6\,\text{N}\cdot\text{m}.\end{aligned}$$

8.78

Both the net force and the net torque exerted on the bridge should vanish for it to be in mechanical equilibrium. For the net force we have

$$+\uparrow \sum F_v = F_{RA} + F_{RB} - F_{W1} - F_{W2} - F_{W3} = 0,$$

where $F_{W1} = 8000\,\text{N}$ is the weight of the automobile on the left, $F_{W2} = 20\,000\,\text{N}$ is that of the one in the middle, and $F_{W3} = 8000\,\text{N}$ is that of the one on the right. For the net torque, we choose point A as the pivot point to obtain

$$\overset{+}{\circlearrowleft} \sum \tau_A = -F_{W1} l_1 + F_{W2} l_2 + F_{W3} l_3 - F_{RB} l_4 = 0,$$

where $l_1 = 15\,\text{m}$, $l_2 = 35\,\text{m}$, $l_3 = 35\,\text{m} + 15\,\text{m} = 50\,\text{m}$, and $l_4 = 35\,\text{m} + 15\,\text{m} + 20\,\text{m} = 70\,\text{m}$. Solve these two equations for F_{RA} and F_{RB}. The results are $F_{RA} = 22\,\text{kN}$ and $F_{RB} = 14\,\text{kN}$.

8.87

Consider the torques exerted on the hammer about its front edge. The torque of the 300-N force is clockwise and has a lever arm of 25 cm; while that of F_N, the force exerted by the nail, is counterclockwise with a lever arm of 9.0 cm. Thus

$$\overset{+}{\circlearrowleft} \sum \tau = (300\,\text{N})(25\,\text{cm}) - F_N(9.0\,\text{cm}) = 0,$$

which yields $F_N = 0.83\,\text{kN}$. From Newton's Third Law the magnitude of the force acting on the nail is also 0.83 kN.

8.88

The net torque about the pivot point of the forceps where the two cutting blades are hinged together is zero. Thus if we denote the force exerted by the rubber on the forceps to be F_r, then $(10.0\,\text{N})(9.0\,\text{cm}) = F_r(3.0\,\text{cm})$, which gives $F_r = 30\,\text{N}$. Again, this is the same as the magnitude of the force the forceps exerts on the rubber, in accordance with Newton's Third Law.

To find F_P, the force exerted on the pivot point by each forceps, consider the balance of forces for each forceps. This gives $10\,\text{N} + 30\,\text{N} - F_P = 0$, so $F_P = 40\,\text{N}$.

8.95

The two blocks are identical, so from symmetry considerations the *c.g.* of the two-block system should be at the center of the glued portion of the two blocks, a distance $\frac{1}{2}(12\,\text{cm}+4\,\text{cm}) = 8\,\text{cm}$ from either end of the system.

8.99

Since the suspended body is in mechanical equilibrium, the three forces exerted on it must sum up to zero: $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_{T1} + \vec{\mathbf{F}}_{T2} + \vec{\mathbf{F}}_{W} = 0$. Here $\vec{\mathbf{F}}_{T1}$ is the force exerted by Scale-1, $\vec{\mathbf{F}}_{T2}$ is that exerted by Scale-2, and $\vec{\mathbf{F}}_{W}$ is the weight of the body. The horizontal component of the vector equation above reads

$$\overset{+}{\rightarrow}\sum F_x = F_{T2}\cos 60.0^\circ - F_{T1}\cos 60.0^\circ = 0\,,$$

which gives $F_{T2} = F_{T1} = 100\,\text{N}$. To find the weight of the body, write down the vertical component of the sum-of-force equation:

$$+\!\uparrow\sum F_y = F_{T1}\sin 60.0^\circ + F_{T2}\sin 60.0^\circ - F_W = 0\,,$$

so $F_W = F_{T1}\sin 60.0^\circ + F_{T2}\sin 60.0^\circ = 2(100\,\text{N})(\sin 60.0^\circ) = 173\,\text{N}$.

The line-of-action of each of the two tension forces intersect at a point directly below the *c.g.* This is because $\vec{\mathbf{F}}_W$ must pass through the same point to make the net torque on the body zero.

8.100

First, find the weight of the triangle by balancing the torque exerted on the bottom rod: $F_{WF}\,(40.0\,\text{cm}) = F_{W\triangle}(20.0\,\text{cm})$, or $F_{W\triangle} = (10.0\,\text{N})(40.0\,\text{cm})/(20.0\,\text{cm}) = 20.0\,\text{N}$. Now balance the torque exerted on the top rod. The clockwise torque is $F_{WO} \times 50.0\,\text{cm}$; while the counterclockwise torque is $(10.0\,\text{N} + 20.0\,\text{N})(30.0\,\text{cm})$. Equate the magnitude of these two torques and solve for F_{WO}, the weight of the sphere:

$$F_{WO} = \frac{(10.0\,\text{N} + 20.0\,\text{N})(30.0\,\text{cm})}{50.0\,\text{cm}} = 18.0\,\text{N}\,.$$

8.103

Let the length of each block be L and its mass be m. For maximum extension the *c.g.* of the topmost block is right above the edge of the one beneath it. Similarly, the combined *c.g.* of the top two blocks, x_{cg}, should be right above the edge of the third one. Thus, measured from the left end of the topmost block, $x_{cg} = (mL + mL/2)/2m = 3L/4$. Similarly, for the top three blocks $x_{cg} = (2mL + mL/2)/3m = 5L/6$; and for the top four $x_{cg} = (3mL + mL/2)/4m = 7L/8$. The overhangs of the first through the fourth blocks are therefore $L/2$, $L/4$, $L/6$, and $L/8$, the sum of which is

$$\frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{8} = \frac{25}{24}L > L\,,$$

meaning that the topmost block does indeed have its entire length *beyond* the edge of the bottom one!

8.109

The net torque and force exerted on the woman must both vanish for her to be in balance. Let the forces exerted on her hands (H) and feet (F) be F_H and F_F, respectively. Then, measured about her *c.g.*,

$$\circlearrowleft^{+} \sum \tau = F_H(0.50\,\text{m}) - F_F(1.00\,\text{m}) = 0$$

and

$$+\uparrow \sum F_y = F_H + F_F - F_W = 0 .$$

Plug in $F_W = (65\,\text{kg})(9.81\,\text{m/s}^2) = 638\,\text{N}$ and solve for F_H and F_F to obtain $F_H = 0.42\,\text{kN}$ and $F_F = 0.22\,\text{kN}$. This means that the force on each hand is 0.21 kN, and that on each foot is 0.11 kN.

8.116

The moment-of-inertia of a uniform, solid cone of mass m and base radius R about its symmetry axis is $I_C = \frac{3}{10}mR^2$. In our case two such cones are present, each with $m = 3.0\,\text{kg}$ and $R = 0.30\,\text{m}$. So the total moment-of-inertia is

$$I = 2I_C = \frac{3}{5}MR^2 = \frac{3}{5}(3.0\,\text{kg})(0.30\,\text{m})^2 = 0.16\,\text{kg}\cdot\text{m}^2 .$$

8.132

The moment-of-inertia of a hoop of mass m and radius R about its central symmetry axis is $I = mR^2$. Thus from Eq. (8.37), $\sum \tau_0 = I\alpha$, we may obtain the resulting angular acceleration α of the hoop:

$$\alpha = \frac{\sum \tau_0}{mR^2} = \frac{0.60\,\text{N}\cdot\text{m}}{(2.0\,\text{kg})(0.50\,\text{m})^2} = 1.2\,\text{rad/s}^2 .$$

8.157

The total KE of the ball is the sum of its translational and rotational KE. For the translational (T) part $\text{KE}_T = \frac{1}{2}mv^2$, and for the rotational (R) part $\text{KE}_R = \frac{1}{2}I\omega^2$. Here $I = \frac{2}{5}mR^2$, with R being the radius of the ball. Also, $v = \omega R$ for pure rolling without slipping. Thus

$$\text{KE}_{\text{total}} = \text{KE}_T + \text{KE}_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}mv^2 .$$

8.163

Apply the Conservation of Angular Momentum: $L_i = I_i\omega_i = L_f = I_f\omega_f$. As ω_f increases by 450%, I_f must *decrease* by 450%.

8.168

The size of Jupiter is negligible compared with its orbital radius around the Sun. So we may think of it as just a point mass, to which Eq. (8.42) applies: $L_0 = r_\perp m_\bullet v$, to calculate the orbital angular momentum L_0 of Jupiter. Here $m_\bullet = 1.9 \times 10^{27}$ kg, $r_\perp = 7.8 \times 10^{11}$ m, and $v = v_{av} = 13.1 \times 10^3$ m/s. Thus

$$L_0 = rm_{\bullet\perp}v_{av} = (1.9 \times 10^{27}\,\text{kg})(7.8 \times 10^{11}\,\text{m})(13.1 \times 10^3\,\text{m/s}) = 1.9 \times 10^{43}\,\text{kg}\cdot\text{m}^2/\text{s}.$$

Now compute the spin angular momentum $L_\odot$ of the Sun using Eq. (8.44): $L_\odot = I_\odot \omega_\odot$. Taking the Sun to be a uniform, solid sphere of mass $M_\odot$ and radius $R_\odot$, then $I_\odot = \frac{2}{5} M_\odot R_\odot^2$. Also, $\omega_\odot = 2\pi/T_\odot$, where $T_\odot$ is the period of rotation of the Sun, which is $(26\,\text{d})(86\,400\,\text{s/d}) = 2.246\,4 \times 10^6$ s. Thus

$$\begin{aligned} L_\odot = I_\odot \omega_\odot &= \left(\frac{2}{5} M_\odot R_\odot^2\right)\left(\frac{2\pi}{T_\odot}\right) = \frac{4\pi M_\odot R_\odot^2}{5T_\odot} \\ &= \frac{4\pi(1.99 \times 10^{30}\,\text{kg})(6.96 \times 10^8\,\text{m})^2}{5(2.246\,4 \times 10^6\,\text{s})} \\ &= 1.1 \times 10^{42}\,\text{kg}\cdot\text{m}^2/\text{s}, \end{aligned}$$

which is less than 10% of the orbital angular momentum of Jupiter.