PHYSICS 1A - CHAPTER 6

6.5

Use Eq. (6.3): $W = Fs\cos\theta$. Here both the force and the displacement are horizontal so $\theta = 0$. Thus

$$W = Fs\cos\theta = (15\,\mathrm{N})(1.5\,\mathrm{m})(\cos 0) = 23\,\mathrm{J}\,,$$

which is done in overcoming friction.

6.9

Again, use Eq. (6.3). Here $F = 20\,\mathrm{N}$, $s = 10\,\mathrm{cm} = 0.10\,\mathrm{m}$, and $\theta = 0$. Thus

$$W = Fs\cos\theta = (20\,\mathrm{N})(0.10\,\mathrm{m})(\cos 0) = 2.0\,\mathrm{J}\,.$$

6.11

Solve for the average force F from Eq. (6.3): $F = W/s\cos\theta$. In this case $W = 400\,\mathrm{J}$, $s = 100\,\mathrm{m}$, and $\theta = 0$; so

$$F = \frac{W}{s\cos\theta} = \frac{400\,\mathrm{J}}{100\,\mathrm{m}(\cos 0)} = 4.00\,\mathrm{N}\,.$$

6.21

Since the force $\vec{\mathbf{F}}$ in question is along the y-direction the work it does on the point mass as the mass undergoes an infinitesimal displacement $d\vec{\mathbf{s}} = (dx)\,\hat{\mathbf{i}} + (dy)\,\hat{\mathbf{j}} + (dz)\,\hat{\mathbf{k}}$ is

$$dW = \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = F_y\,\hat{\mathbf{j}}\cdot\left[(dx)\,\hat{\mathbf{i}} + (dy)\,\hat{\mathbf{j}} + (dz)\,\hat{\mathbf{k}}\right] = F_y\,dy\,.$$

Plug in $F_y = (5.0\,\mathrm{N/m})y$ and integrate over y from $y_i = 0$ to $y_f = 20.0\,\mathrm{m}$ to obtain

$$W = \int_{y_i}^{y_f} F_y\,dy = \int_0^{20.0\,\mathrm{m}} (5.0\,\mathrm{N/m})y\,dy = (5.0\,\mathrm{N/m})\left[\frac{y^2}{2}\right]_0^{20.0\,\mathrm{m}} = 1.0\,\mathrm{kN\cdot m}\,.$$

6.25

The weight of the object is being supported by five segments of rope, as shown in Fig. P25. Each segment supports 1/5 of the weight. The force that is needed to pull out the rope, being equal to the tension in the rope, is therefore only 1/5 of the weight to be raised. Since work-in = work-out, in order to raise the weight by 1.0 m we need to pull out $5 \times 1.0\,\mathrm{m} = 5.0\,\mathrm{m}$ of the rope.

6.26

The direction of $\vec{\mathbf{F}}$, the force pulling the barge, makes an angle of $\theta = 30°$ with the direction of motion of the barge. So the work done against friction is

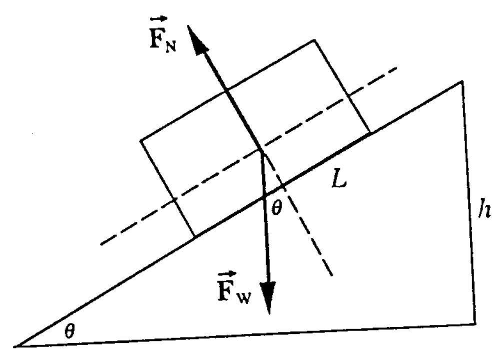
$$W = Fs\cos\theta = (1000\,\mathrm{N})(10\times10^3\,\mathrm{m})(\cos 30°) = 8.7\times10^6\,\mathrm{J} = 8.7\,\mathrm{MJ}\,.$$

6.27

The net upward displacement of the person was $h = (30\,\mathrm{stairs})(25.0\,\mathrm{cm/stair}) = 750\,\mathrm{cm} = 7.50\,\mathrm{m}$. The force he exerted in pulling himself up was equal to his body weight of $F_W = mg = (59.1\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 579.8\,\mathrm{N}$. So the net work he did was

$$W = Fh = (579.8\,\mathrm{N})(7.50\,\mathrm{m}) = 4.35\,\mathrm{kJ}\,.$$

6.29



The force that must be applied on the box (whose weight is F_W) to push it up the incline at a constant speed is $F = F_W\sin\theta$, where θ is the angle the incline makes with the horizontal. Let the length of the incline be L, then the work done in pushing the box to the top of the incline is $W = F_\parallel L = (F_W\sin\theta)L = F_W(L\sin\theta) = F_W h$, where $h = L\sin\theta = 5.0\,\mathrm{m}$ is the height of the top of the incline (see the figure to the right). Hence

$$W = F_W h = (100\,\mathrm{N})(5.0\,\mathrm{m}) = 5.0\times10^2\,\mathrm{J}\,.$$

This is the same as the work that has to be done in lifting the box straight up by 5.0 m.

6.30

Let the total weight of the crates plus the wagon be F_W. In order to push the load up an incline, a force $F = F_W\sin\theta$ needs to be applied on the load to overcome gravity. Here θ is the angle of inclination. The work done by F as the load is pushed up the incline by a distance s is then $W = Fs = F_W s\sin\theta$. With $F_W = 400\,\mathrm{N} + (10.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 498.1\,\mathrm{N}$, $s = 10.0\,\mathrm{m}$, and $\theta = 30.0°$,

$$W = F_W s\sin\theta = (498.1\,\mathrm{N})(10.0\,\mathrm{m})(\sin 30.0°) = 2.49\,\mathrm{kJ}\,.$$

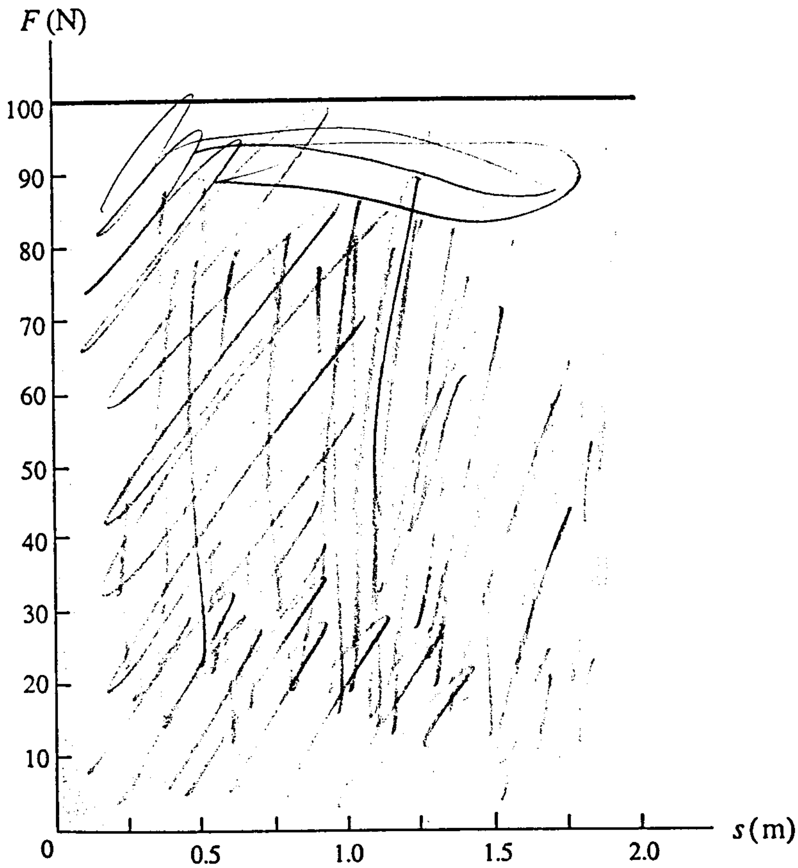
6.35

(a) The total work done is $W = Fs\cos\theta = (100\,\mathrm{N})(2.0\,\mathrm{m})(\cos 0) = 2.0 \times 10^2\,\mathrm{J} = 0.20\,\mathrm{kJ}$.

(b) Since the scales of the F vs s diagram are $1.0\,\mathrm{cm} = 10\,\mathrm{N}$ for F and $1.0\,\mathrm{cm} = 0.25\,\mathrm{m}$ for s, an area of $1.0\,\mathrm{cm}^2$ on the diagram corresponds to $(1.0\,\mathrm{cm}^2)(10\,\mathrm{N}/1.0\,\mathrm{cm})(0.25\,\mathrm{m}/1.0\,\mathrm{cm}) = 2.5\,\mathrm{J}$ of work.

(c) The area under the F vs s curve is $A = (10\,\mathrm{cm} \times 8.0\,\mathrm{cm})(2.5\,\mathrm{J/cm^2}) = 2.0 \times 10^2\,\mathrm{J}$, which is equal to the work W computed in part (a) above.

The force versus displacement plot is shown below. The area of the shaded portion in the plot represents the amount of work done by the force F over a distance s.



6.37

The work done is equal to the area under the F vs s curve. From $x = 2\,\mathrm{m}$ to $x = 6\,\mathrm{m}$ the corresponding area is $W = (20\,\mathrm{N})(6\,\mathrm{m} - 2\,\mathrm{m}) = 0.8 \times 10^2\,\mathrm{J} = 0.08\,\mathrm{kJ}$, and from $x = 0$ to $x = 10\,\mathrm{m}$ the corresponding area is

$$W = \frac{1}{2}(2\,\mathrm{m})(20\,\mathrm{N}) + (20\,\mathrm{N})(6\,\mathrm{m} - 2\,\mathrm{m}) + \frac{1}{2}(10\,\mathrm{m} - 6\,\mathrm{m})(20\,\mathrm{N}) = 1.4 \times 10^2\,\mathrm{J} = 0.14\,\mathrm{kJ}.$$

Since the average force was the least in magnitude from $x = 9\,\mathrm{m}$ to $x = 10\,\mathrm{m}$, the least amount of work was done over that 1-m interval. After $x = 10\,\mathrm{m}$ the force became negative, so the work done from $x = 10\,\mathrm{m}$ on was also negative. Thus from $x = 2\,\mathrm{m}$ to $x = 12\,\mathrm{m}$

$$\begin{aligned} W &= (20\,\mathrm{N})(6\,\mathrm{m} - 2\,\mathrm{m}) + \frac{1}{2}(20\,\mathrm{N})(10\,\mathrm{m} - 6\,\mathrm{m}) + \frac{1}{2}(-20\,\mathrm{N})(11\,\mathrm{m} - 10\,\mathrm{m}) \\ &\quad + (-20\,\mathrm{N})(12\,\mathrm{m} - 11\,\mathrm{m}) \\ &= 0.9 \times 10^2\,\mathrm{J} = 0.09\,\mathrm{kJ}. \end{aligned}$$

6.54

Use Eq. (6.13), with $m = 1.0\,\text{g} = 1.0 \times 10^{-3}\,\text{kg}$ and $v = 70\,\text{km/s} = 70 \times 10^3\,\text{m/s}$:

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(1.0 \times 10^{-3}\,\text{kg})(70 \times 10^3\,\text{m/s})^2 = 2.5 \times 10^6\,\text{J} = 2.5\,\text{MJ}.$$

6.56

If the final speed of the spaceship is v_f, then its final kinetic energy is $\text{KE}_f = \frac{1}{2}mv_f^2$, where m is the mass of the spaceship. This energy comes from the nuclear energy of the uranium, so $\text{KE}_f = \frac{1}{2}mv_f^2 = \text{E}_U = 7.4 \times 10^{16}\,\text{J}$. Solve for v_f:

$$v_f = \sqrt{\frac{2\text{KE}_f}{m}} = \sqrt{\frac{2\,\text{E}_U}{m}} = \sqrt{\frac{2(7.4 \times 10^{16}\,\text{J})}{3.5 \times 10^6\,\text{kg}}} = 2.1 \times 10^5\,\text{m/s}.$$

6.79

The increase in gravitational potential energy of a load of mass m as it ascends by a vertical displacement h is $\Delta\text{PE}_G = mgh$. This comes from the chemical energy $\text{PE}_C = 6 \times 10^9\,\text{J}$. Let $\Delta\text{PE}_G = mgh = \text{PE}_C$ and solve for h:

$$h = \frac{\text{PE}_C}{mg} = \frac{6 \times 10^9\,\text{J}}{(1 \times 10^6\,\text{kg})(9.8\,\text{m/s}^2)} = 6 \times 10^2\,\text{m}.$$

6.87

With Problem (6.83) in mind, we note that here ΔPE_G is negative since the car is headed downhill (so $\Delta h < 0$). Hence

$$\begin{aligned}\Delta\text{KE} + \Delta\text{PE}_G &= \frac{1}{2}mv^2 + mg\Delta h \\ &= \frac{1}{2}(1000\,\text{kg})(20\,\text{m/s})^2 + (1000\,\text{kg})(9.81\,\text{m/s}^2)(-100\,\text{m}) \\ &= -7.8 \times 10^5\,\text{J} = -0.78\,\text{MJ}.\end{aligned}$$

6.89

The weight of George-the-monkey is $F_W = 10\,\text{N}$, which is much less than the weight attached to the other end of the rope. So unless the monkey accelerates up the rope at an incredible rate of $a = 9g$ or greater, which is unrealistic, he will not be able to lift the 100-N weight off the floor. In the following, we will assume that he moves up the rope at a constant speed, so $a = 0$.

(a) Since we assume that the monkey climbs up the rope at a constant speed there is no change in his kinetic energy. The work he does is equal to the change in his *gravitational*-PE: $W = F_W\Delta h = (10\,\text{N})(10\,\text{m}) = 1.0 \times 10^2\,\text{J} = 0.10\,\text{kJ}$.

(b) Since the 100-N weight is not lifted off the floor, the rope does not move, so no rope ends up on the floor.

(c) The 100-N mass does not move, so there is no change in its *gravitational*-PE. So for the system $\Delta\text{PE}_G = 0.10\,\text{kJ}$ [see part (b) above)], which is due to the motion of the monkey alone.

6.101

Consider one of the two cars, which has a mass m and is moving at an initial speed v_i towards the hill. Its kinetic energy before climbing the hill is $KE_i = \frac{1}{2}mv_i^2$. Suppose that the greatest height of a hill the car can successfully climb is h_{max} provided that the car does not lose any energy to friction. Then the car can barely make it to the top of such a hill, meaning that by the time it does so it must have no KE left, i.e. all of its initial KE has been converted to its final *gravitational*-PE. Thus $KE_i = \frac{1}{2}mv_i^2 = PE_{Gf} = mgh_{max}$, which gives the maximum height h_{max} of the hill it can climb to be

$$h_{max} = \frac{v_i^2}{2g}.$$

Note that this result is *independent* of the mass m of the car. In our case, with $v_i = (96\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s}) = 26.7\,\text{m/s}$, the maximum scalable height is

$$h_{max} = \frac{(26.7\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 36.2\,\text{m}.$$

As this is greater than the 33.5-m height each car has to climb, both cars should be able to make it to the top of the hill.

6.112

Use conservation of energy for the space probe of mass m: $E_i = E_f$, where $E_i = KE_i + PE_{Gi} = \frac{1}{2}mv_i^2 - GmM_\oplus/R_\oplus = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_{esc}^2$ is the initial mechanical energy (kinetic plus gravitational potential energy) for the probe as it is just launched from the surface of the Earth; and $KE_f = KE_f + PE_{Gf} = \frac{1}{2}mv_f^2$ is its final mechanical energy as it is practically infinitely far away from the Earth (where the *gravitational*-PE due to the Earth's gravitational field is set to be zero). Note that $v_{esc} = \sqrt{2GM_\oplus/R_\oplus}$, given by Eq. (6.24), leads to the result $GmM_\oplus/R_\oplus = \frac{1}{2}mv_{esc}^2$, which we used in rewriting E_i. Thus

$$E_i = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_{esc}^2 = E_f = \frac{1}{2}mv_f^2.$$

Solve for v_i, the required initial speed of the probe:

$$v_i = \sqrt{v_f^2 + v_{esc}^2} = \sqrt{(5.00\,\text{km/s})^2 + (11.2\,\text{km/s})^2} = 12.3\,\text{km/s}.$$