10.5

The work W done on the spring is equal to the gain in its elastic potential energy:

$$W = \Delta \mathrm{PE}_{e} = \frac{1}{2}ks^2 = \frac{1}{2}(2.00 \times 10^2\,\mathrm{N/m})(0.100\,\mathrm{m})^2 = 1.00\,\mathrm{J}\,.$$

10.15

Apply the Principle of Conservation of Energy. Before hitting the spring, the object of mass m moving at speed v has a kinetic energy of $\mathrm{KE} = \frac{1}{2}mv^2$, where $m = 2.00\,\mathrm{kg}$ and $v = 1.50\,\mathrm{m/s}$. Suppose it compresses the spring for a distance s before coming to a momentary stop. Then all of its initial KE is converted into *elastic*-PE upon stopping: $\mathrm{KE} = \frac{1}{2}mv^2 = \mathrm{PE}_{e} = \frac{1}{2}ks^2$. Solve for s:

$$s = v\sqrt{\frac{m}{k}} = (1.50\,\mathrm{m/s})\sqrt{\frac{2.00\,\mathrm{kg}}{25.0\,\mathrm{N/m}}} = 0.424\,\mathrm{m} = 4.24\,\mathrm{cm}\,.$$

10.16

When a spring is stretched or compressed by an amount x it exerts a force $F = -kx$, where k is its spring constant. Assuming that this force is conservative, then

$$\Delta \mathrm{PE}_{e} = \mathrm{PE}_{ef} - \mathrm{PE}_{ei} = -\int_{P_i}^{P_f} F(x)\,dx = -\int_{x_i}^{x_f} -kx\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\,.$$

Thus $\mathrm{PE}_{e}(x) = \frac{1}{2}kx^2$.

10.19

The force F stretching the rod is the weight of the mass m: $F = F_W = mg$. The cross-sectional area of the rod is $A = \pi R^2$, where $R = 0.707\,\mathrm{mm}$. Thus from Eq. (10.4)

$$\sigma = \frac{F}{A} = \frac{mg}{\pi R^2} = \frac{(4.00\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{\pi(0.707 \times 10^{-3}\,\mathrm{m})^2} = 2.50 \times 10^7\,\mathrm{Pa}\,.$$

10.33

Use Eq. (10.7): $Y = \sigma/\varepsilon$. Here $Y = 50\,\mathrm{GPa}$ for marble; and $\sigma = 110\,\mathrm{MPa}$, which corresponds to ε_R, the maximum strain. Thus

$$\varepsilon_R = \frac{\sigma}{Y} = \frac{110 \times 10^6\,\mathrm{Pa}}{50 \times 10^9\,\mathrm{Pa}} = 2.2 \times 10^{-3} = 0.22\%\,.$$

10.41

Use Eq. (10.7): $Y = \sigma/\varepsilon$. Here $\sigma = 0.050\,\mathrm{MPa}$ and $Y = 200\,\mathrm{GPa}$ for structural steel. Thus the percentage of its compression is

$$\varepsilon = \frac{\sigma}{Y} = \frac{0.50\,\mathrm{MPa}}{200 \times 10^3\,\mathrm{MPa}} = 2.5 \times 10^{-6} = 2.5 \times 10^{-4}\%\,.$$

10.51

The maximum load F_{max} applied on the cable of cross-sectional area A is $F_{max} = 25\,000\,\text{lb} \times (4.454\,\text{N/lb}) = 1.113 \times 10^5\,\text{N}$, which causes a stress of $\sigma = F_{max}/A = F/(\frac{1}{4}\pi D^2)$, with D the diameter of the cable. To stay below the yield strength σ_{max} of the cable, let $\sigma \leq \sigma_{max} = F_{max}/A = F_{max}/(\frac{1}{4}\pi D^2_{min})$, which gives D_{min}, the required minimum diameter of the cable:

$$D_{min} = \sqrt{\frac{4F_{max}}{\pi\sigma_{max}}} = \sqrt{\frac{4(1.113 \times 10^5\,\text{N})}{\pi(345 \times 10^6\,\text{Pa})}} = 0.0203\,\text{m} = 2.03\,\text{cm}\,.$$

The corresponding maximum strain is then obtained from Eq. (10.7):

$$\varepsilon_{max} = \frac{\sigma_{max}}{Y} = \frac{345\,\text{MPa}}{200\,\text{GPa}} = 1.73 \times 10^{-3} = 0.173\%\,.$$

10.59

The period T is the time it takes for one complete revolution. Since the record makes $33\frac{1}{3}$ revolutions per minute,

$$T = \frac{(1\,\text{min})(60\,\text{s/min})}{33\frac{1}{3}} = 1.8\,\text{s}\,.$$

10.61

The motion of the ant as seen by the child is a one-dimensional SHM, which is the circular motion of the ant projected onto a line perpendicular to the line-of-sight of the child. The maximum displacement of the ant either to the left or the right measured from the center of

the motion, which coincides with the center of the record, is R, the radius of the record. Since the record turns at 78 rpm, the period of the motion is $T = (1/78)$ min, and the corresponding frequency is

$$f = \frac{1}{T} = \frac{78}{(1\text{min})(60\,\text{s/min})} = 1.3\,\text{Hz}\,.$$

According to Eq. (10.12) the angular frequency is $\omega = 2\pi f = (2\pi\,\text{rad})(1.3\,\text{s}^{-1}) = 8.2\,\text{rad/s}$.

10.63

The acceleration as a function of time of an object in SHM is given by Eq. (10.17): $a_x = -A\omega^2 \cos\omega t$. For maximum acceleration $a_x(\text{max})$, set $\cos\omega t = \pm 1$ to obtain $|a_x(\text{max})| = A\omega^2 = A(2\pi f)^2$. With $A = 0.50\,\text{cm}$ and $f = 50\,\text{Hz}$,

$$|a_x(\text{max})| = 4\pi^2 A f^2 = 4\pi^2(0.50 \times 10^{-2}\,\text{m})(50\,\text{Hz})^2 = 4.9 \times 10^2\,\text{m/s}^2\,.$$

10.67

Compare the expression $x = 5.0\cos(0.40t + 0.10)$ for the SHM of the body in question with Eq. (10.13), $x = A\cos\omega t = x_{max}\cos 2\pi ft$, which describes a standard SHM with an initial displacement of A.

(a) $A = 5.0\,\text{m}$, by direct comparison between the two equations.

(b) Set $2\pi f = 0.40\,\text{s}^{-1}$ to obtain $f = 0.40\,\text{s}^{-1}/2\pi = 0.064\,\text{Hz}$.

(c) Set $t = 0$ in the expression for the phase to obtain the initial phase: $\varepsilon = 0.40 \times 0 + 0.10 = 0.10\,\text{rad}$.

(d) Plug $t = 2.0\,\text{s}$ into the expression for x: $x = 5.0\cos\left[(0.40\,\text{s}^{-1})(2.0\,\text{s}) + 0.10\right] = 3.1\,\text{m}$.

10.69

Since $d\cos(\omega t)/dt = -\omega\sin\omega t$,

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(x_{max}\cos 2\pi ft) = -x_{max}(2\pi f)\sin 2\pi ft\,.$$

10.85

The weight of the potatoes of mass m is $F_W = mg$, which is the force F applied on the scale. The resulting displacement of the scale is $x = 2.50\,\text{cm} = 0.0250\,\text{m}$; hence from $F_W = mg = F = kx$ we get

$$k = \frac{mg}{x} = \frac{(2.00\,\text{kg})(9.81\,\text{m/s}^2)}{0.0250\,\text{m}} = 785\,\text{N/m}\,.$$

The frequency f_0 of the SHM is found from Eq. (10.20):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{785\,\text{N/m}}{2.00\,\text{kg}}} = 3.15\,\text{Hz}\,.$$

10.93

The weight of the object of mass m is $F_W = mg$ which, when applied to the spring, results in an elongation of ΔL. Thus the spring constant k is given by $k = mg/\Delta L$. The period T of the subsequent SHM then follows from Eq. (10.20):

$$T = \frac{1}{f_0} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/\Delta L}}\,.$$

This gives an expression for g, in terms of measured quantities T and ΔL:

$$g = \frac{4\pi^2\Delta L}{T^2}\,.$$

10.101

The period of a simple pendulum is given by $T = 1/f_0 = 2\pi\sqrt{L/g}$. Plug in $T = 10.0\,\text{s}$ and solve for L, the length of the pendulum:

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.81\,\text{m/s}^2)(10.0\,\text{s})^2}{4\pi^2} = 24.8\,\text{m}\,.$$

10.107
As the mass m moves away from the equilibrium position at the origin by an amount x, the restoring force exerted on it by the spring is $F = -kx$, where the minus sign indicates that the force is opposite to the direction of the displacement. According to Newton's Second Law this would lead to an acceleration $a = d^2x/dt^2$ for the spring:

$$F = -kx = ma = m\frac{d^2x}{dt^2},$$

or

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0.$$

10.113
The period of a simple pendulum of length L is given by $T = 1/f_o = 2\pi\sqrt{L/g}$. When the length is increased by 50%, to $L' = L + 50\%L = 1.5L$, the new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{1.5L}{g}} = \sqrt{1.5}\left(2\pi\sqrt{\frac{L}{g}}\right) = \sqrt{1.5}\,T = 1.2\,T.$$