Theory of Fine-Scale Zonal Flow Generation

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Motivation

• The RH neoclassical polarization $1.6 \varepsilon^{3/2} q_r^2 \rho_{\theta i}^2$


• RH neoclassical polarization expression is valid for $q_r \rho_{\theta i} \ll 1$.
Motivation

• A relatively short scale ZF \((q_r \rho_{\theta i} \sim 1)\) has been found in Nonlinear GTC simulation of Trapped Electron Mode (TEM) turbulence [Z. Lin et al., IAEA-CN/TH/P2-8 (2006) & Y. Xiao, et al., TTF Meeting (2008)].

This figure of Zonal flow length scale for CTEM is from Yong Xiao’s work.
Turbulence driven radial current shielded by ZF

\[ en_0 \chi_{total} \frac{\partial}{\partial t} \left( \frac{e\phi_z}{T_e} \right) + \frac{\partial}{\partial r} \left( \frac{\delta \langle J_r \rangle}{\delta \phi_z} \phi_z \right) = 0 \] [Diamond et al., IAEA (1998)]

\( \phi_z \) zonal flow potential, \( \langle J_r \rangle \) transport-induced radial current
\( \chi_{total} \) polarization including neoclassical enhancement

- Poynting theorem for drift wave turbulence

\[ J_r = -\frac{n_0 T_e}{2} \sum_k \frac{ck_\theta}{B} \frac{\partial}{\partial r} \left( \frac{\partial \chi}{\partial \phi_k} \right) \left| \frac{e\phi_k}{T_e} \right|^2 \] [Diamond and Kim, PFB 3, 1626 (1991)]

\( \chi \) : susceptibility

- Use the definition of wave action density \( N \)

\[ N = \frac{E_k}{\omega_k} = -\frac{n_0 T_e}{2} \frac{\partial \chi}{\partial \omega} \left| \frac{e\phi_k}{T_e} \right|^2 \] in wave kinetic approach

[Lebedev et al., PoP 2, 4420 (1995)]
ZF Generation and Random Shearing Occur Simultaneously

- $\omega_k \gg \Omega_z \rightarrow$ Drift Wave action density, $N$, is conserved (adiabatic invariant)
- Random Shearing by Zonal Flows results in increase of $k_r$ of DWT. [Diamond et al., IAEA ’98 & Hahm et al., PoP ‘99]
- With $E_k = N\omega_k$, DW Energy decreases as $\omega_k$ is down-shifted.
- $\frac{d}{dt}(E_{DW} + E_{ZF}) = 0 \rightarrow$ ZF growth [Diamond et al., IAEA ‘98]
ZF Growth Rate from Wave Kinetic Approach

- Wave kinetic equation [Diamond et al., IAEA (1998)]

\[
\frac{\partial}{\partial t} N + \left( \tilde{v}_g + \tilde{v}_{ZF} \right) \cdot \frac{\partial}{\partial \tilde{x}} N - \frac{\partial}{\partial \tilde{x}} \left( \omega + \tilde{k} \cdot \tilde{v}_{ZF} \right) \cdot \frac{\partial}{\partial \tilde{k}} N = \gamma_k N - \frac{\Delta \omega_k}{N_0} N^2
\]

- Using scale separation and action response to ZF variation, \( \tilde{N} \), we obtain the ZF growth rate

\[
\gamma_z = -\frac{1}{\chi_{total}} q_r c_s^2 \rho_s^2 \sum_k k_\theta^2 \frac{\partial \chi}{\partial k_r} \bigg|_{\omega_k} \frac{2 \partial \langle N \rangle / \partial k_r}{n_0 T_e \partial \chi / \partial \omega} R(k,q_r) \left( 1 - \frac{\chi_z}{\omega_e / \omega_k} \right)
\]

Expect growth \( \gamma_z > 0 \) for \( \partial \langle N \rangle / \partial k_r < 0 \)
Neo-polarization for $q_r \rho_{\theta_i} \sim 1$

- Neo-Polarization from bounce-kinetic equation (via action-angle variables $\rightarrow$ conventional guiding center variables) [Fong and Hahm, PoP 6, 188 (1996)]

- For $q_r \rho_{\theta_i} \ll 1 : \chi_{nc}^{tr} = 1.35 \tau \varepsilon^{3/2} q_r^2 \rho_{\theta_i}^2$
  
  The same scaling with RH expression

- For $q_r \rho_{\theta_i} \gg 1 : \chi_{nc}^{tr} = 0.9 \tau \sqrt{\varepsilon} (1 - \frac{0.2}{\sqrt{\varepsilon} q_r \rho_{\theta_i}})$

- The generalized analytic expression

$$\chi_{total} = \frac{1}{1.6 \varepsilon^{3/2} q_r^2 \rho_{\theta_i}^2} + \frac{1}{0.9 \sqrt{\varepsilon} (1 + \frac{0.2}{\sqrt{\varepsilon} q_r \rho_{\theta_i}})} + \tau (1 - 0.9 \sqrt{\varepsilon} (1 - \Gamma_0 (\varepsilon q_r \rho_{\theta_i})))$$

  strongly circulating particles

  barely circulating particles
Results (Neo-polarization)

For arbitrary $q_r \rho_{\theta i}$ Residual zonal flow $\beta = \chi_{cl} / \chi_{total}$

The left figure is taken from Y. Xiao’s PhD thesis (2006): Fig 4-5 (numerical)
The right one is our generalized analytic result.

Our analytic expression agrees with Y. Xiao’s numerical result very well!
ZF Growth Rate for TEM

Using the susceptibility & linear growth rate for CTEM and the generalized polarization

This figure is for \( k_{\perp} \rho_i \sim 0.2 \)

and \( \varepsilon = 0.18 \)

The spatial scale of ZF at which \( \gamma_z / \gamma_{TEM} \) maximizes \( \sim \)ZF scale from Z. Lin (2006) and Y. Xiao (2008)!

Normalized ZF growth rate and characteristic \( q_r \) increase with an increase in electron temperature gradient.
These figures are for \( k_{\perp} \rho_i \sim 0.2 \) and \( \varepsilon = 0.18 \).

Normalized ZF growth rate and characteristic \( q_r \) increase with a decrease in density gradient and temperature ratio \( T_e / T_i \).
Conclusion

- Generalized neoclassical polarization expression for full banana width $q_r \rho_{\theta i} \sim 1$ is obtained from bounce-kinetic equation.
- ZF Growth for short spatial scale is obtained.
- Qualitatively consistent with GTC simulation results (Yong’s Job)

- Open questions:
  - ZF saturation mechanism still needs to be addressed.
  - Role of trapped electron nonlinearity [previously concluded to be small in Lin et al., IAEA (2004)]
  - Robustness of WKE approach for short scale ZF
Thank you!