Theory of Fine-Scale Zonal Flow Generation

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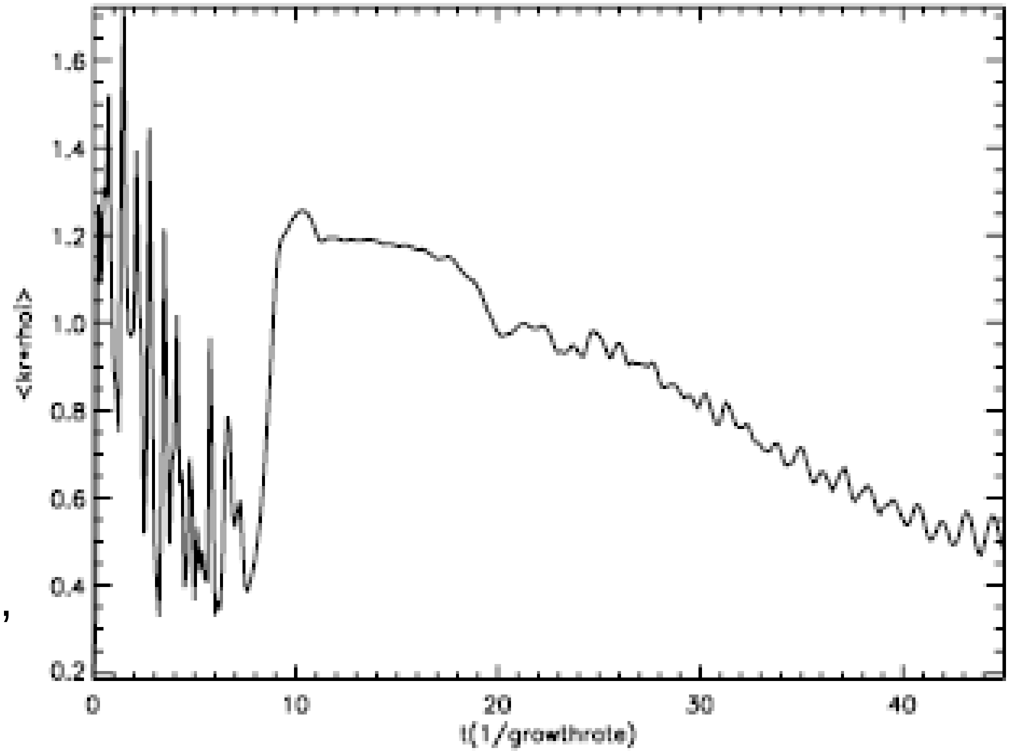
Motivation

- The RH neoclassical polarization $1.6\varepsilon^{3/2} q_r^2 \rho_{\theta i}^2$ [Rosenbluth and Hinton, PRL 80, 724 (1998)] was used in most ZF generation theories [P. H. Diamond et al., IAEA-CN-69/TH3/1 (1998) & L. Chen et al., PoP 7, 3129 (2000)].

- RH neoclassical polarization expression is valid for $q_r \rho_{\theta i} \ll 1$.

Motivation

- A relatively short scale ZF $(q_r \rho_{\theta i} \sim 1)$ has been found in Nonlinear GTC simulation of Trapped Electron Mode (TEM) turbulence [Z. Lin et al., IAEA-CN/TH/P2-8 (2006) & Y. Xiao, et al., TTF Meeting (2008)].



This figure of Zonal flow length scale for CTEM is from Yong Xiao's work

Turbulence driven radial current shielded by ZF

$$en_0\chi_{total}\frac{\partial}{\partial t}\left(\frac{e\phi_z}{T_e}\right)+\frac{\partial}{\partial r}\left(\frac{\delta\langle J_r\rangle}{\delta\phi_z}\phi_z\right)=0$$ [Diamond et al., IAEA (1998)]

ϕ_z zonal flow potential, $\langle J_r\rangle$ transport-induced radial current

χ_{total} polarization including neoclassical enhancement

- Poynting theorem for drift wave turbulence

$$J_r=-\frac{n_0T_e}{2}\sum_k\frac{ck_\theta}{B}\frac{\partial}{\partial r}\left(\left.\frac{\partial\chi}{\partial k_r}\right|_{\omega_k}\left|\frac{e\phi_k}{T_e}\right|^2\right),$$ χ : susceptibility

[Diamond and Kim, PFB 3, 1626 (1991)]

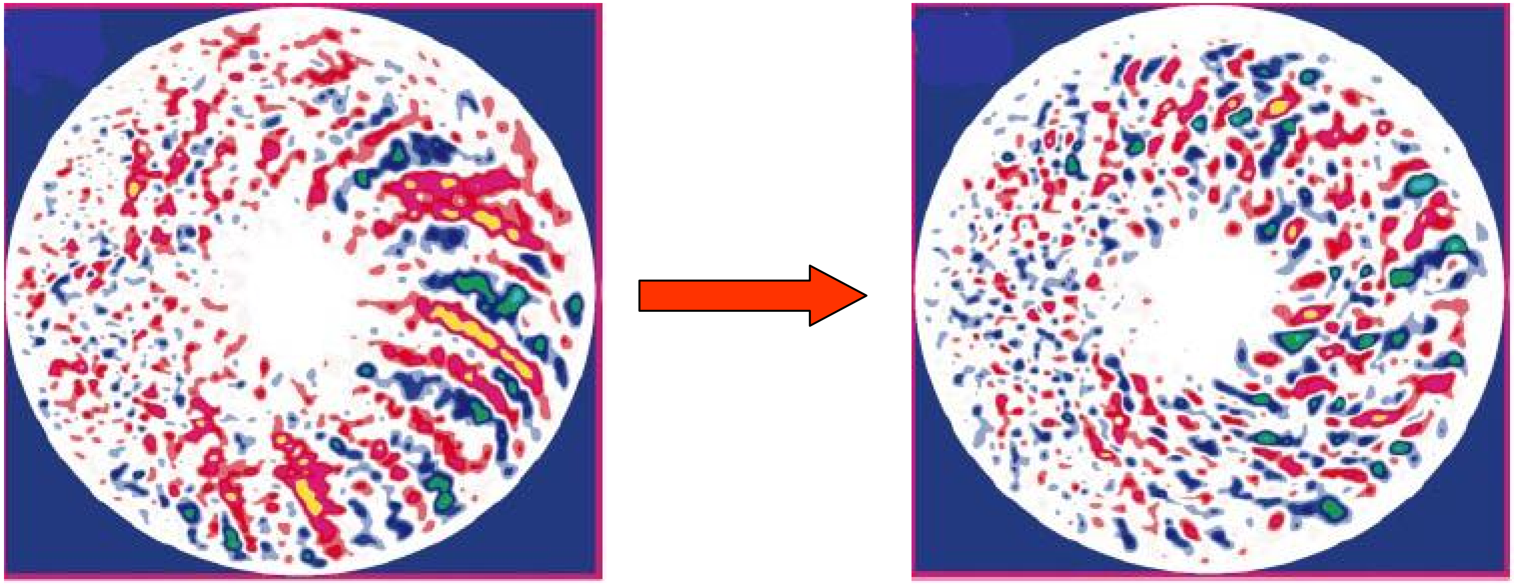
- Use the definition of wave action density N

[Lebedev et al., PoP 2, 4420 (1995)]

$$N=\frac{E_k}{\omega_k}=-\frac{n_0T_e}{2}\frac{\partial\chi}{\partial\omega}\left|\frac{e\phi_k}{T_e}\right|^2$$ in wave kinetic approach

ZF Generation and Random Shearing Occur Simultaneously

- $\omega_k \gg \Omega_z \rightarrow$ Drift Wave action density, N , is conserved (adiabatic invariant)
- Random Shearing by Zonal Flows results in increase of k_r of DWT. [Diamond et at., IAEA '98 & Hahm et al., PoP '99]
- With $E_k = N\omega_k$, DW Energy decreases as ω_k is down-shifted.
- $\frac{d}{dt}\left(E_{DW} + E_{ZF}\right) = 0, \rightarrow$ ZF growth [Diamond et al., IAEA '98]



ZF Growth Rate from Wave Kinetic Approach

- Wave kinetic equation [Diamond et al., IAEA (1998)]

$$\frac{\partial}{\partial t}N+\left(\vec{v}_g+\vec{v}_{ZF}\right)\cdot\frac{\partial}{\partial\vec{x}}N-\frac{\partial}{\partial\vec{x}}\left(\omega+\vec{k}\cdot\vec{v}_{ZF}\right)\cdot\frac{\partial}{\partial\vec{k}}N=\gamma_k N-\frac{\Delta\omega_k}{N_0}N^2$$

- Using scale separation and action response to ZF variation, $\tilde{N}$, we obtain the ZF growth rate

$$\gamma_z=-\frac{1}{\chi_{total}}q_r^4c_s^2\rho_s^2\sum_k k_\theta^2\left.\frac{\partial\chi}{\partial k_r}\right|_{\omega_k}\frac{2\partial\langle N\rangle/\partial k_r}{n_0T_e\partial\chi/\partial\omega}R\left(k,q_r\right)\left(1-\frac{\chi_z}{\omega_{*e}/\omega_k}\right)$$

Expect growth $\gamma_z>0$ for $\partial\langle N\rangle/\partial k_r<0$

Neo-polarization for $q_r \rho_{\theta i} \sim 1$

- Neo-Polarization from bounce-kinetic equation (via action-angle variables→ conventional guiding center variables) [Fong and Hahm, PoP 6, 188 (1996)]

- For $q_r \rho_{\theta i} \ll 1$: $\chi_{nc}^{tr} = 1.35 \tau \varepsilon^{3/2} q_r^2 \rho_{\theta i}^2$

  The same scaling with RH expression

- For $q_r \rho_{\theta i} \gg 1$: $\chi_{nc}^{tr} = 0.9 \tau \sqrt{\varepsilon} (1 - \frac{0.2}{\sqrt{\varepsilon} q_r \rho_{\theta i}})$

- The generalized analytic expression

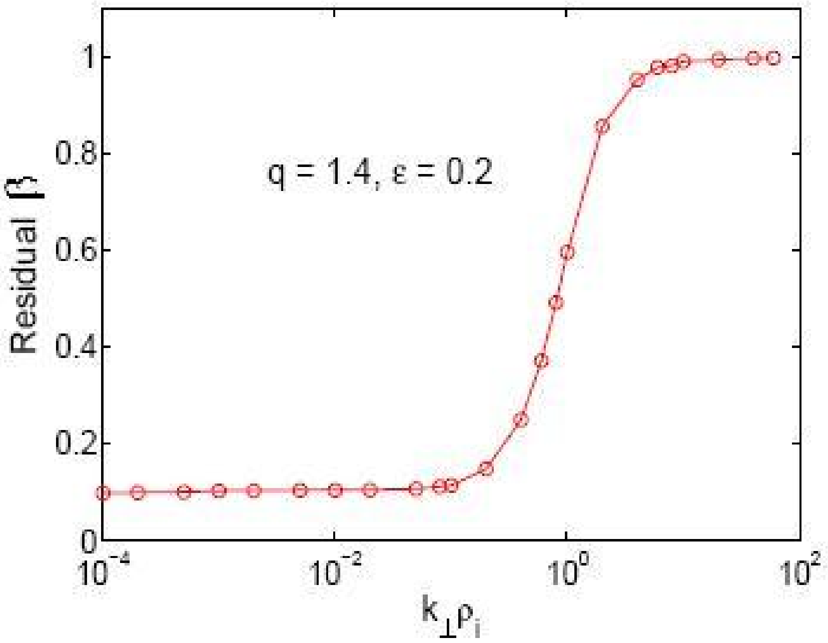
$$\chi_{total} = \frac{\tau}{\frac{1}{1.6 \varepsilon^{3/2} q_r^2 \rho_{\theta i}^2} + \frac{1}{0.9\sqrt{\varepsilon}} (1 + \frac{0.2}{\sqrt{\varepsilon} q_r \rho_{\theta i}})} + \tau (1 - 0.9 \sqrt{\varepsilon}) \left(1 - \Gamma_0 (\varepsilon q_r \rho_{\theta i})\right)$$

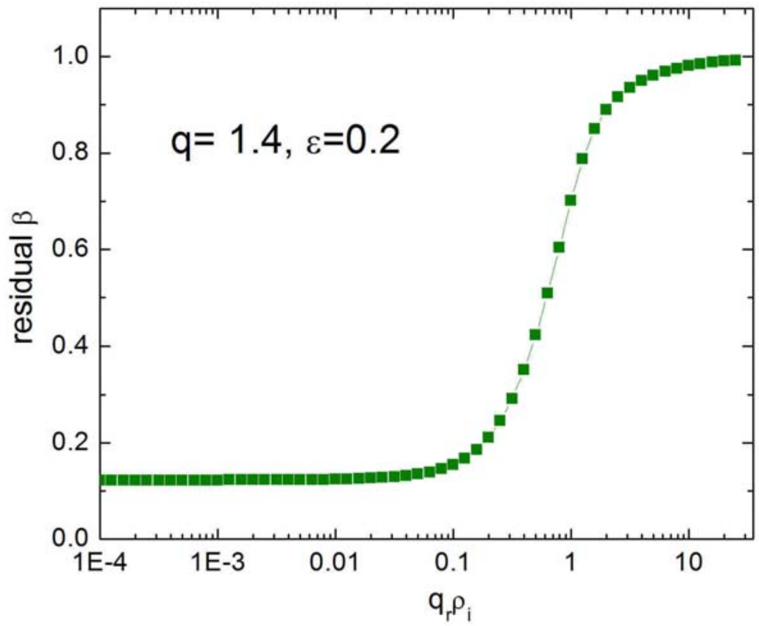
strongly circulating particles

barely circulating particles

Results (Neo-polarization)

For arbitrary $q_r \rho_{\theta i}$ Residual zonal flow $\beta = \chi_{cl} / \chi_{total}$

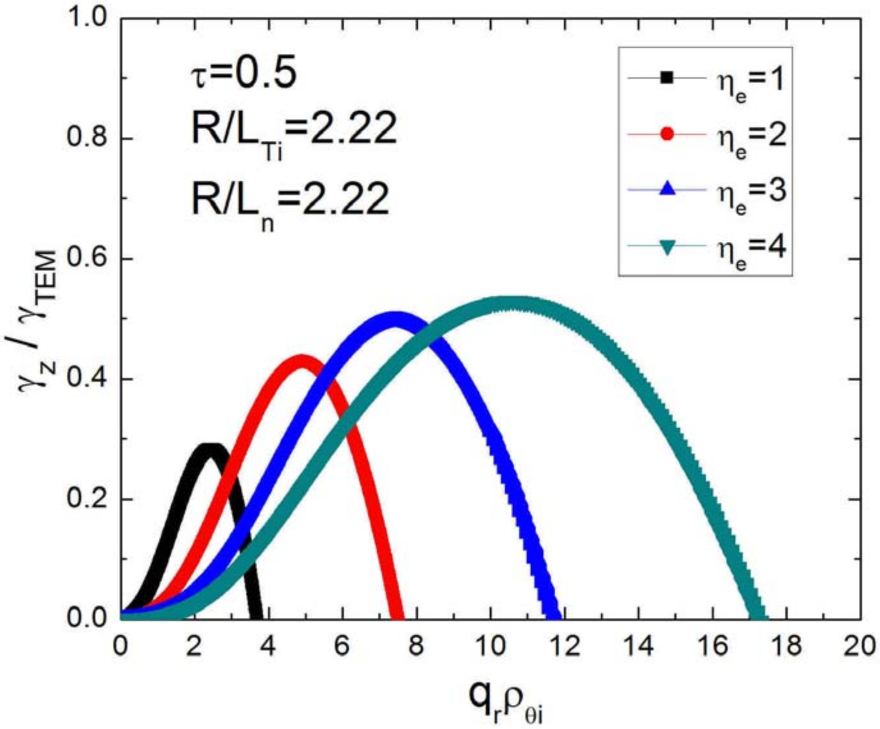




The left figure is taken from Y. Xiao's PhD thesis(2006): Fig 4-5 (numerical)
The right one is our generalized analytic result.

Our analytic expression agrees with
Y. Xiao's numerical result very well!

ZF Growth Rate for TEM



Using the susceptibility & linear growth rate for CTEM and the generalized polarization

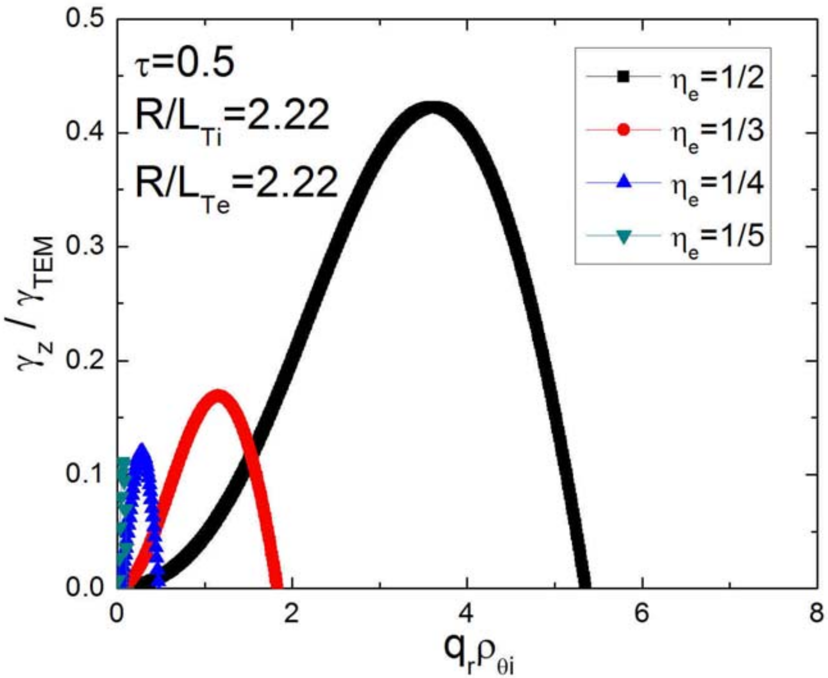
This figure is for $k_\perp \rho_i \sim 0.2$

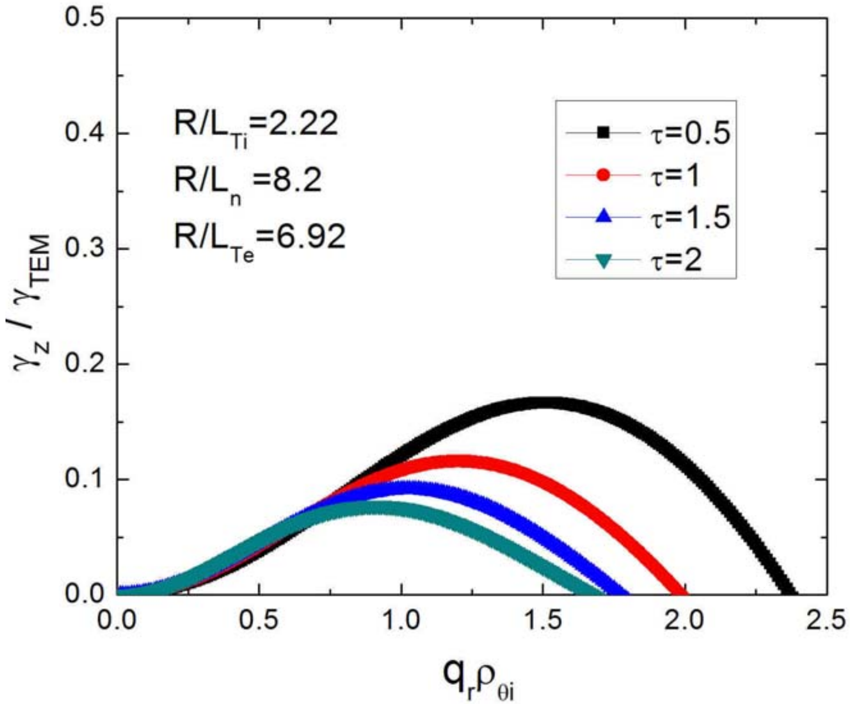
and $\varepsilon = 0.18$

The spatial scale of ZF at which γ_z / γ_{TEM} maximizes $\sim$ZF scale from Z. Lin (2006) and Y. Xiao (2008)!

Normalized ZF growth rate and characteristic q_r increase with an increase in electron temperature gradient.

ZF Growth Rate for TEM





These figures are for $k_{\perp}\rho_i \sim 0.2$ and $\varepsilon = 0.18$

Normalized ZF growth rate and characteristic q_r increase with a decrease in density gradient and temperature ratio T_e / T_i.

Conclusion

- Generalized neoclassical polarization expression for full banana width $q_r \rho_{\theta i} \sim 1$ is obtained from bounce-kinetic equation.
- ZF Growth for short spatial scale is obtained.
- Qualitatively consistent with GTC simulation results (Yong's Job)

- Open questions:
  - ZF saturation mechanism still needs to be addressed.
  - Role of trapped electron nonlinearity [previously concluded to be small in Lin et al., IAEA (2004)]
  - Robustness of WKE approach for short scale ZF

Thank you!