Heliospheric tomography: an algorithm for the reconstruction of the 3D solar wind from remote sensing observations

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ABSTRACT

Over the past years we have developed a tomographic technique for using heliospheric remote sensing observations (*i.e.* interplanetary scintillation and Thomson scattering data) for the reconstruction of the three-dimensional solar wind density and velocity in the inner heliosphere. We describe the basic algorithm on which our technique is based. To highlight the details of the reconstruction algorithm we specifically emphasize the implementation of corotating tomography using IPS *g*-level and IPS velocity observations as proxies for the solar wind density and velocity, respectively. We provide some insight into the modifications required to expand the technique into a fully time-dependent tomography, and to use Thomson scattering brightness (instead of *g*-level) as a proxy for the solar wind density.

Keywords: Tomography, 3D reconstruction, heliosphere, solar wind, space weather

1. INTRODUCTION

Heliospheric remote sensing provides a means for sampling the solar wind plasma over a large range of elongations relative to the Sun. This provides global information about the three-dimensional density and velocity structure of the solar wind in the inner heliosphere, including regions over the solar poles that are difficult to access by any other means. In principle, this provides an opportunity for directly reconstructing the solar wind structure in the inner heliosphere. The main problem that must be addressed is the ambiguity introduced by line-of-sight integration. Each observation is the integrated effect of an unknown distribution of material along the line of sight. Tomography provides a methodology to resolve this ambiguity and reconstruct the distribution of material in three-dimensions in cases where observations from multiple perspectives are available.

We have developed a tomography technique that uses heliospheric remote sensing data and depends on rotation of the Sun (similar to tomography with coronal data[1,2]), but also outward motion of structures in the solar wind to obtain different perspectives of structures embedded in the solar wind. This technique requires continuous observations from a single perspective only, thus making it applicable to several existing remote sensing data sets. We have applied this technique successfully to interplanetary scintillation (IPS) observations[3] and Thomson scattering observations[4,5]. The technique allows us to determine the large-scale three-dimensional extents of solar wind structures, and forecast their arrival at Earth.

In this paper the basic algorithm underlying our tomographic technique is discussed in detail. Here we emphasize the use of IPS observations. These measurements have been used to probe solar wind features with ground-based meter-wavelength radio observations[6,7]. The scintillation is caused by small-scale (~200km) density variations in the solar wind. IPS provides information of the solar wind velocity through correlation of IPS signals from multi-site radio arrays[8,9,10]. The scintillation measured by single-site IPS arrays provides a proxy for the heliospheric density[11,12].

We will also assume that the solar wind density and velocity is static (time-independent) in a reference frame that corotates with the Sun. This restricts the usefulness of this corotating tomography technique to remote sensing data obtained during periods when the solar wind evolves on time scales longer than one Carrington rotation, a condition occurring primarily near solar minimum.

In Section 2 we provide the basic equations underlying the numerical algorithm. In Section 3 the numerical implementation is discussed, including the heliospheric grid structure used, and the discretization of the line-of-sight integrals. The iterative process is described in Section 4, including the kinematic solar wind model that is at the core of this tomography technique. In Section 5 we briefly discuss several possible extensions to this tomography, such as the implementation of a time-dependent version of the technique, and the use of Thomson scattering data instead of *g*-level data as a proxy for the solar wind density.

2. IPS REMOTE SENSING DATA

The observational data used in the tomographic reconstruction are the IPS *g*-level (or 'disturbance factor') and IPS velocity *W* observations. Each observation represents a line-of-sight integration through the solar wind in the inner heliosphere. To date we have used *g*-level data from the IPS arrays in Cambridge (UK), Ooty (India), and more recently Nagoya (Japan); *W* data have been available from Nagoya and Ooty. The purpose of the reconstruction is to create a model 3D heliosphere (*i.e.*, a density and velocity distribution) that reproduces these observations as well as possible. The reconstruction uses an iterative scheme to minimize the differences between actually observed and calculated model values. Since the reconstruction only involves the solar wind density *n* and the (radial) solar wind outflow velocity *V*, the observed quantities *g* and *W* need to be related to these two solar wind parameters.

The *g*-level[13] is related to the scintillation index *m*:

$$g = m / \langle m \rangle \, . \tag{1}$$

m is the instantaneous, observed scintillation index for an IPS source; <*m*> is the expected 'quiet' scintillation index, based on an average of past source observations as a function of solar elongation. *g* depends only weakly on elongation (or heliocentric distance). The scintillation index is defined as an integration along the entire line of sight:

$$m^2 = \int_0^\infty ds \rho(s) \delta n(s)^2 \, . \tag{2}$$

The 'weight' function $\rho(s)$ depends on observing wavelength, the angular size of the radio source, and the turbulence power spectrum[14]. We use the same expression as given in Eq. 2 of Jackson *et al.* (2000)[3] with a power law slope close to 3.0. The small-scale density fluctuations $\delta n(s)$ along the line of sight cause the scintillation. $\delta n(s)$ depends on local plasma properties, not only macroscopic (solar wind speed, density, magnetic field), but also microscopic properties associated with turbulence in the solar wind. However, empirical evidence[12] suggests that changes in $\delta n(s)$ are related to changes in density. Quantitatively we model this behavior by expressing $\delta n(s)$ in terms of the heliocentric distance *r* and the normalized solar wind density $\hat{n} = (r/r_0)^2 n$.

$$\delta n(r, \hat{n}) = \delta n_0 (r_0 / r)^{2-\beta_r} (\hat{n}/n_0)^{\beta_n} \, . \tag{3}$$

The characteristic length scale r_0 = 1 AU. The tomography is run using reasonable values for the powers $\beta_n \approx 0.5$[2,3] and $\beta_r \approx 0$. The mean scintillation index <*m*> (and hence the *g*-level) depends on parameters $\hat{n}^{mean}$ and δn_0^{mean} describing the density and density fluctuation in the 'average' background solar wind.

The IPS velocity is defined in terms of the same weight function and small-scale density fluctuations as the *g*-level:

$$W = \int_0^\infty ds \rho(s) \delta n(s)^2 V^{\perp}(s) \Big/ \int_0^\infty ds \rho(s) \delta n(s)^2 \, . \tag{4}$$

A more accurate calculation of *W* is possible using the cross correlation of the IPS signals at different IPS stations. The expression used here is consistent with the more accurate calculation to within ≈10 km/s, while significantly reducing the required computational resources[15]. $V^{\perp}$ is the component of the solar wind velocity perpendicular to the line of sight at a distance *s* from the observer.

We introduce a few definitions to arrive at the equations for *g* and *W* implemented in the numerical algorithm. First, write the background solar wind density $\hat{n}^{mean}$ (needed to calculate <*m*>) as:

$$\hat{n}^{mean}(s) = n_{mean}\bar{n}_{mean}(s). \tag{5}$$

The constant n_{mean} sets the absolute density scale. $\bar{n}_{mean}$ is a dimensionless function defining the shape of the mean background solar wind density distribution. Also redefine the normalized density $\hat{n}$ to absorb the constants δn_0, δn_0^{mean} and n_{mean}:

$$\eta = \left(\delta n_0 / \delta n_0^{mean}\right)^{1/\beta_n} \hat{n}/n_{mean} . \tag{6}$$

This 'modified normalized density' will be useful later in the discussion of the kinematic solar wind model (Section 4.2). Define the 'γ function':

$$\gamma = \eta^{2\beta_n} . \tag{7}$$

Finally, define a new weight function σ that absorbs the dependence on heliocentric distance:

$$\sigma(s,\beta_r) = \rho(s) r(s)^{-2(2-\beta_r)}. \tag{8}$$

With these definitions the equations for *g*-level and IPS speed *W* become:

$$\begin{aligned} g^2 &= \int_0^\infty ds\,\sigma(s,\beta_r)\gamma(s) \Big/ \int_0^\infty ds\,\sigma(s,\beta_r)(\bar{n}_{mean}(s))^{2\beta_n} \\ W &= \int_0^\infty ds\,\sigma(s,\beta_r)\gamma(s)V(s)\sin\chi(s) \Big/ \int_0^\infty ds\,\sigma(s,\beta_r)\gamma(s) \end{aligned} \tag{9}$$

where we put $V^{\perp}(s) = V(s)\sin\chi(s)$. $\chi(s)$ is the angle between the direction to the Sun and to the Earth from the position at distance *s* along the line of sight. In Eq. (9) we usually set $\bar{n}_{mean} = 1$, *i.e.* we are assuming a background solar wind density with a $1/r^2$ drop-off.

There are three unknown functions in these equations: γ (or equivalently the normalized density $\hat{n}$), the radial solar wind outflow speed *V*, and $\bar{n}_{mean}$, the shape of the background normalized solar wind density. The first two, γ and *V*, are the ones that we are interested in. The reconstruction problem can be formulated as follows: for a given shape $\bar{n}_{mean}$ (specified over the heliospheric volume of interest), and for a given set of *g*-level and *W* observations, find the functions γ and V that best fits these observations.

Several points can be made about Eq. (9):

1. g^2 is a more 'natural' observational quantity than *g* itself. Both g^2 and *W* are described in terms of very similar integrals, with $\gamma(s)$ and $V(s)\sin\chi(s)$ specifying the contribution of a line-of-segment at distance *s* from the observer to the observed *g*-level and IPS speed, respectively.

2. The nominator and denominator for *g* and *W* have the same dependence on heliocentric distance, hence the *g*-level and velocity *W* will both be nearly independent of solar elongation.

3. Both n_{mean} and β_n have disappeared. Their influence has been reduced to determining the density scale (Eq. 6) together with the unknown constants δn_0 and δn_0^{mean}. The solution for γ and V depends explicitly[•] only on β_r, which controls the weighting for the line-of-sight contribution $\gamma(s)$ and $V(s)\sin\chi(s)$.

4. For a given shape $\overline{n}_{mean}$, if the pair of functions $\hat{n}, V$ is the solution for a specific n_{mean}, then the pair $\alpha\hat{n}, V$ is the solution for αn_{mean} for any constant α.

The last point above implies that for a given set of *g*-level and *W* observations the solution for the normalized density $\hat{n}$ is determined only up to constant, *i.e.*, the absolute density scale of the solution cannot be determined from the tomographic reconstruction itself. This is not surprising: the *g*-level, our proxy for the solar wind density, is only a relative statement about the state of the solar wind along the line of sight to the IPS source as compared with 'average' conditions (Eq. 1). What these 'average' conditions are, must be established using external information. For instance, the density scale can be calibrated against solar wind densities observed *in situ* at 1 AU near Earth. This external calibration defines the relationship between γ and $\hat{n}$ in Eqs. (6) and (7), and hence also implicitly defines the constants δn_0 and δn_0^{mean}.

3. THE NUMERICAL ALGORITHM

The reconstruction task has been reduced to finding the functions γ and V for a given set of *g*-level and *W* observations satisfying Eq. 9. The normalized density $\hat{n}$ follows from γ using Eqs. (6) and (7).

3.1. Notation: use of subscripts

In the following the subscripts i, j, k are used when a quantity refers to the 3D heliospheric grid (next subsection) used to compute heliospheric γ-function and velocity, and for quantities at the source surface (when only i, j will appear). The subscript μ is used to identify a line of sight; while ν refers to a segment at a certain distance from the observer along the line of sight.

3.2. The Remote-Sensing Observations

The reconstruction is based on a set of N_{obs} line-of-sight observations for both *g*-level and IPS velocity *W*. For simplicity we assume that there are as many *g*-level data as there are *W* data (this will not be the case if *g*-level and velocity are from different observing locations):

$$\left(g^2\right)_\mu^{obs}, W_\mu^{obs} \qquad (\mu = 1, N_{obs}) . \tag{10}$$

'Model' observations are calculated from the latest iterative 3D model of heliospheric γ-function and velocity *V*, and are compared with the actual observations. The purpose of the reconstruction is to create a model 3D heliosphere that matches these observations as closely as possible. The 'model observations' are given by

$$\left(g^2\right)_\mu^{mdl}, W_\mu^{mdl} \qquad (\mu = 1, N_{obs}) . \tag{11}$$

Each line of sight is subdivided in N_{los} segments of length ds_μ. The segment length is expressed as a constant in units of the Sun-Earth distance. Since for Earth-based observations this distance varies with time, and observations are taken

[•] However, in Section 4.2 we will see that β_n is still needed in the solar wind model where the implementation of mass and mass flux conservation require the calculation of the modified normalized density η from γ using Eq. 7.

at different times, this means that the segment length is different for each line of sight). The distance from observer to the center of each line segment is given by:

$$s_{\nu,\mu} = (\nu - 0.5) ds_\mu \qquad (\nu = 1, N_{los};\quad \mu = 1, N_{obs})\,. \tag{12}$$

Typical values are $ds_\mu \approx 0.05$ and $N_{los} = 40$, so that each line of sight extends about 2 AU out from the observer.

3.3. The Solar Wind Plasma

The grid used in the reconstruction is regular in heliographic longitude, heliographic latitude, and heliocentric distance, and is fixed relative to the Sun (*i.e.* rotates with the sidereal solar rotation rate, P_{sid}). The range of longitudes covered by the grid (360°, *i.e.* a whole solar rotation), is expressed in terms of the 'Carrington variable', c. One unit in Carrington variable covers 360° in heliographic longitude. The integer part, $\mathrm{int}(c)$, is a conventional Carrington rotation number, and effectively sets the time period of interest. The fraction is related to the heliographic longitude, $\lambda = 360^\circ \times \{1 - (c - \mathrm{int}(c))\}$

The range of heliographic longitudes for the grid is set by a start and end 'Carrington variable': c_{beg} and c_{end}. Note that the grid does not need to start at 0°, *i.e.* at the exact start of a new Carrington rotation. Associated with the variables c_{beg} and c_{end} are the times, t_{beg} and t_{end} at which the corresponding heliographic longitude crossed the center of the solar disk as seen from Earth (or, more general, 'the observer'). These times determine which observations are used for the reconstruction. *E.g.* all g-level and W observations inside the time interval [t_{beg} , t_{end}] are included.

The latitude grid covers the full range –90° to +90°. The radial grid covers the range from the 'source surface' at R_s to some outer boundary at $R_{\max}$.

$$\begin{aligned}
c_i &= c_{beg} + (i-1)\times dC \qquad & dC &= (c_{end} - c_{beg})/(N_c - 1) \qquad & (i &= 1, N_c)\\
l_j &= -\pi/2 + (j-1)\times dL \qquad & dL &= \pi/(N_l - 1) \qquad & (j &= 1, N_l)\\
r_k &= R_s + (k-1)\times dR \qquad & dR &= (R_{\max} - R_s)/(N_r - 1) \qquad & (k &= 1, N_r)
\end{aligned} \tag{13}$$

Typical values for the grid parameters are R_s = 15 solar radii, R_{max} = 3 AU, dR = 0.1 AU, dC = 1/36 (*i.e.* 10° in heliographic longitude), and dL = 10°.

The tomography reconstructs the 3D heliospheric γ function (and its associated normalized density $\hat{n}$) and velocity V in the grid points of Eq. 13:

$$\gamma_{i,j,k},\; V_{i,j,k} \qquad (i = 1, N_c;\, j = 1, N_l;\, k = 1, N_r) \tag{14}$$

The lower boundary of the heliospheric grid (k=1), the 'source surface', plays a crucial role in the tomographic reconstruction. The γ function and velocity at the source surface are:

$$\gamma_{i,j}^{source} = \gamma_{i,j,k=1} \qquad V_{i,j}^{source} = V_{i,j,k=1} \qquad (i = 1, N_c;\, j = 1, N_l) \tag{15}$$

Specifying the content of these two maps initializes the reconstruction; each iteration is completed by updating them.

3.4. Discretization of the line of sight integrals

The calculation of model g-level and IPS velocity W observations requires an integration through the model heliosphere along the same directions (lines of sight) as the actual observations (Eq. 9). In discrete from this becomes:

$$\left(g^2\right)_\mu^{mdl} = \sum_{\nu=1}^{N_{los}} H_{\nu,\mu}\gamma_{\nu,\mu} \qquad\qquad W_\mu^{mdl} = \sum_{\nu=1}^{N_{los}} \mathrm{K}_{\nu,\mu} V_{\nu,\mu} \sin\chi_{\nu,\mu} \tag{16}$$

with weight factors

$$H_{\nu,\mu} = \sigma_{\nu,\mu} \Big/ \sum_{\nu=1}^{N_{los}} \sigma_{\nu,\mu} \qquad\qquad \mathrm{K}_{\nu,\mu} = \sigma_{\nu,\mu}\gamma_{\nu,\mu} \Big/ \sum_{\nu=1}^{N_{los}} \sigma_{\nu,\mu}\gamma_{\nu,\mu} \tag{17}$$

where

$$\sigma_{\nu,\mu} = \sigma\left(s_{\nu,\mu}\right) = \rho\left(s_{\nu,\mu}\right) r_{\nu,\mu}^{-2(2-\beta_r)}. \tag{18}$$

The μ-dependence of $s_{\nu,\mu}$ enters through the μ-dependence of ds_μ (Eq. 12). The function $\rho(s)$ does not depend on the line of sight orientation (*i.e.* the elongation), but does depend on the distance along the line of sight.

The heliospheric γ function and velocity in all line-of-sight segments

$$\gamma_{\nu,\mu}, V_{\nu,\mu} \qquad\qquad (\nu = 1, N_{los};\ \ \mu = 1, N_{obs}) \tag{19}$$

are obtained by linear interpolation on the 3D γ function and velocity (Eq. 14) at the center positions (Eq. 12) of all line of sight segments.

4. THE ITERATIVE PROCESS

4.1. Brief Outline

The iterative process is started by specifying γ function values and velocity V at the source surface (Eq. 15). Using these source surface values the 3D γ function values and velocities (Eq. 14) in the heliospheric grid (Eq. 13) is obtained by applying a solar wind model for the propagation of mass from the source surface out into the heliosphere. We assume radial outflow and apply simple kinematic arguments to conserve mass and mass flux (Section 4.2). At this stage also ‘traceback’ information is accumulated which connects each heliospheric grid point to its ‘source location’ at the source surface.

Model line of sight observations (Eq. 11) are calculated by integrating along the appropriate directions through the 3D heliosphere. These model values are compared with the actual observations (Eq. 10). This comparison provides the main convergence criterion for the iterative process. The observed-to-model ratios for all lines of sight will be used to determine the source surface update, completing the iteration.

All line-of-sight segments are projected back to the source surface using the ‘traceback’ information, carrying along the observed-to-model ratio of the line of sight they belong to. At the source surface all the segments of all lines of sight are assigned to the nearest grid point. The γ function and velocity in the grid point is then updated by combining observed-to-model ratios of all line of segments assigned to the grid point.

We now follow the main steps in this process in detail.

4.2. The Kinematic Solar Wind Model

Given are the γ function values and velocities on the source surface at heliocentric distance $r_0 = R_s$ (Eq. 15). From these we need to reconstruct the γ function values and velocities at ‘higher levels’, *i.e.* heliocentric distances r_k $\left(k = 2, N_l\right)$. The problem is solved by induction.

Given the γ values and velocities at level *k*, the γ values and velocity at level *k+1* need to be determined. The connection between the levels is established using simple kinematic arguments based on conservation laws. Currently we use conservation of mass and mass flux (though other choices, such as conservation of momentum, are easily implemented). Each grid point *i,j* on level *k* (Eq. 14) represents a parcel of mass with a modified normalized density $\eta_{i,j,k}$ (related to $\gamma_{i,j,k}$ through Eq. 7) and velocity $V_{i,j,k}$. The parcel of mass is assumed to travel radially outward at the local speed. When it arrives at level *k+1* it has a modified normalized density $\tilde{\eta}_{i,j,k}$ and velocity $\tilde{V}_{i.j,k}$. The conservation laws for the parcels of mass are (mass and mass flux, respectively) are:

$$\eta_{i,j,k} = \tilde{\eta}_{i,j,k}$$
$$\eta_{i,j,k} V_{i,j,k} = \tilde{\eta}_{i,j,k} \tilde{V}_{i,j,k} \,. \tag{20}$$

In the corotating heliographic coordinate system the parcel will have moved to a larger Carrington variable (smaller heliographic longitude). The parcel will arrive at level *k+1* at Carrington variable:

$$\tilde{c}_{i,j,k} = c_{i,j,k} + (r_{k+1} - r_k)/V_{i,j,k} P_{sid} = c_{i,j}, k + dR/V_{i,j,k} P_{sid} \tag{21}$$

where P_{sid} is the sidereal rotation period of the Sun. Note that the parcel only shifts in longitude, not in heliographic latitude. The position $\tilde{c}_{i,j,k}$ will be located somewhere in between two grid points at level *k+1*. Let these grid points be $(\tilde{i}_{near}, j)$ and $(\tilde{i}_{far}, j)$, where $\left|\tilde{i}_{near} - \tilde{i}_{far}\right| = 1$. Let grid point $(\tilde{i}_{near}, j)$ be closest to $\tilde{c}_{i,j,k}$, and define the difference in Carrington variable $\delta c_{i,j,k} = \left|\tilde{c}_{i,j,k} - c_{\tilde{i}_{near}, j,k}\right|$. At level *k+1* each parcel of mass is split up in two fractions, which are assigned to the neighboring grid points $(\tilde{i}_{near}, j)$ and $(\tilde{i}_{far}, j)$. A fraction $f_{near} = 1 - \delta c_{i,j,k}$ is assigned to $(\tilde{i}_{near}, j)$, and the remaining fraction $f_{far} = \delta c_{i,j,k}$ is assigned to $(\tilde{i}_{far}, j)$. Modified normalized density and velocity at each grid point (i, j), at level *k+1* are obtained by collecting all parcel fractions that have been assigned to it. In terms of the conservation laws:

$$\eta_{i,j,k+1} = \sum_s f_s \tilde{\eta}_{s,j,k} = \sum_s f_s \eta_{s,j,k}$$
$$\eta_{i,j,k+1} V_{i,j,k+1} = \sum_s f_s \tilde{\eta}_{s,j,k} \tilde{V}_{s,j,k} = \sum_s f_s \eta_{s,j,k} V_{s,j,k} \tag{22}$$

Solving for η and *V*:

$$\eta_{i,j,k+1} = \sum_s f_s \eta_{i,j,k}$$
$$V_{i,j,k+1} = \sum_s f_s \eta_{i,j,k} V_{i,j,k} \Big/ \sum_s f_s \eta_{i,j,k} \,. \tag{23}$$

Note that the procedure leading up to Eq. 23 implies that the total mass and mass flux present at a given latitude is the same at all heights.

The displacements $\tilde{c}_{i,j,k} - c_{i,j,k}$ from Eq. 21 are used to construct a 'traceback matrix' S which connects each heliospheric grid point to its origin at the source surface (*i.e.* the point on the source surface which lies on the same stream line as the grid point):

$$c_{i,j,k}^{source} = c_{i,j,k} + S_{i,j,k} \tag{24}$$

This 'traceback matrix' is needed in the final phase of the iteration to project the line-of-sight observations to the source surface (Section 4.4).

4.3. Convergence Criterium and Rejection of Outliers

Improving the model from one iteration to the next is based on a comparison of model observations (Eq. 11) and actual observations (Eq. 10). Error estimates for *g*-levels and IPS velocities *W* are defined as:

$$\sigma_g^2 = N_{obs}^{-1} \sum_{\mu=1}^{N_{obs}} \left(\tau_\mu^g - 1\right)^2$$
$$\sigma_W^2 = N_{obs}^{-1} \sum_{\mu=1}^{N_{obs}} \left(\tau_\mu^W - 1\right)^2 \tag{25}$$

where we defined the ratios of observed and model values:

$$\tau_\mu^g = \left(g^2\right)_\mu^{obs} \Big/ \left(g^2\right)_\mu^{mdl}$$
$$\tau_\mu^W = W_\mu^{obs} \Big/ W_\mu^{mdl} \,. \tag{26}$$

These quantities (Eq. 25), estimates of the relative deviation of model values and actual observations, are the best convergence criteria available. These should move closer to zero from iteration to iteration.

The relative differences of model and actual observations for individual lines of sight (normalized to the 'average deviation' for all sources; Eq. 25)

$$\delta g_\mu = \left(\tau_\mu^g - 1\right) \Big/ \sigma_g$$
$$\delta W_\mu = \left(\tau_\mu^W - 1\right) \Big/ \sigma_W \tag{27}$$

are used as a criterion to flag individual observations as bad. If after a specified number of iterations the relative difference for an observation is above a certain threshold (typically set to 3 for both *g*-level and *W* observations) this is used to justify throwing out the data point. The iterative process is then restarted with these 'outlier' data points removed.

4.4. Projection to the Source Surface

To finish the iteration the results from Section 4.3 need to be translated to the source surface. Let the heliographic coordinates of the line of sight segments (Carrington variable, heliographic latitude, and heliocentric distance, respectively) be:

$$c_{\nu,\mu}, l_{\nu,\mu}, r_{\nu,\mu} \,. \tag{28}$$

This location is 'traced back' to the source surface using the 'traceback matrix' S (Eq. 24). The traceback value at the line of sight segment $S_{\nu,\mu}$ is calculated from a linear interpolation on neighboring heliospheric grid points. The source location is then defined by:

$$c_{\nu,\mu}^{source} = c_{\nu,\mu} + S_{\nu,\mu}; \qquad l_{\nu,\mu}^{source} = l_{\nu,\mu}; \qquad r_{\nu,\mu}^{source} = R_s \,. \tag{29}$$

4.5. Source Surface Updates

The projected locations (Eq. 29) of all line of sight segments will be scattered across the source surface. For each grid element (i, j) at the source surface all segments are collected, located within one half grid spacing of the grid element:

$$c_{i,j} - \tfrac{1}{2} dC \le c_{\nu,\mu}^{source} \le c_{i,j} + \tfrac{1}{2} dC; \qquad l_{i,j} - \tfrac{1}{2} dL \le l_{\nu,\mu}^{source} \le l_{i,j} + dL \,. \tag{30}$$

The ratios in Eq. 26 for these line-of-sight segments are then used to update the source surface γ function and velocity. The group of segments near bin (i, j) is defined by a group of pairs:

$$\left(\nu_a, \mu_a\right) \qquad a = 1, N_{segments}^{i,j} \,. \tag{31}$$

The ratios for this group of segments are combined in a weighted mean to obtain a correction factor to the source surface values:

$$\gamma_{i,j}^{new} \Big/ \gamma_{i,j}^{old} = \sum_{a=1}^{N_{segments}^{i,j}} \mathrm{H}_{\nu_a,\mu_a} \tau_{\mu_a}^{g} \Bigg/ \sum_{a=1}^{N_{segments}^{i,j}} \mathrm{H}_{\nu_a,\mu_a}$$

$$V_{i,j}^{new} \Big/ V_{i,j}^{old} = \sum_{a=1}^{N_{segments}^{i,j}} \mathrm{K}_{\nu_a,\mu_a} \tau_{\mu_a}^{W} \Bigg/ \sum_{a=1}^{N_{segments}^{i,j}} \mathrm{K}_{\nu_a,\mu_a} \quad . \tag{32}$$

I.e., for each segment included in the sum the correction factor τ is weighted proportional to the weight it had in the calculation of the model observation.

Before continuing with the next iteration the new source surface values are smoothed by applying a spatial averaging across the entire source surface. The purpose of this is mainly to 'dampen' the solution, and thereby improve the stability of the iteration process.

5. ADAPTATIONS AND EXTENSIONS

The tomographic algorithm formulated in this paper uses IPS remote sensing data (g-level and IPS velocity) to model a time-independent solar wind (except for corotation) using a very simple kinematic solar wind model to describe the outflow of the solar wind.

Several adaptations are possible (some of which have already been implemented) to overcome these limitations, and add to the capabilities of the technique.

5.1. Corotating Tomography in Forecast Mode

In the definition of the spatial grid (Section 3.3) it was tacitly assumed that $c_{end} - c_{beg} = 1$, *i.e.* that the longitudinal grid extends over exactly 360°. It is also assumed that

$$\gamma_{1,j,k} = \gamma_{N_c,j,k}\,, \qquad V_{1,j,k} = V_{N_c,j,k}\,, \tag{33}$$

i.e. γ function and velocity at opposite ends of the grid (360° apart in heliographic longitude) are the same, as one would expect. In 'normal mode' operation this is indeed strictly enforced.

Near the ends of the grid (near c_{beg} and c_{end}) line-of-sight segments from observations obtained one synodic rotation period of the Sun apart (near t_{beg} and t_{end}) will contribute to the same grid points at the source surface. In 'forecast mode' these contributions are separated by setting $c_{end} - c_{beg} = 3$, defining a longitudinal grid that covers three consecutive Carrington rotations. The symmetry condition Eq. 33 is no longer enforced. In this way a solution is obtained in which grid points at larger Carrington variable are dominated by observations from later times. No mixing of observations taken one Carrington rotation apart occurs. This provides a simple means of studying slow evolution of the quiet solar wind over time scales of one solar rotation within the context of a strictly time-independent technique.

We use this 'forecast mode' to run a real-time space weather forecast project. IPS observations taken by the IPS array operated by STELab in Nagoya, Japan, are received at UCSD within one day, and are processed running the corotating tomography in forecast mode. A time series at Earth is extracted from the solution. This time series is then used to forecast the arrival of corotating structures in the solar wind several days ahead of time (see http://cassfos02.ucsd.edu/solar/index_v_n.html).

5.2. The Solar Wind Model

The kinematic solar wind model currently used has the advantage that is simple and computationally cheap, while obeying very basic conservation laws. Obviously more realistic descriptions of the solar wind are possible. The solar wind model has been intentionally implemented in a modular fashion: the inputs are density and velocity at the source surface, the outputs are the 3D solar wind density and velocity, and the 'traceback' matrix. In principle any solar wind model can be adapted to replace the kinematic model. We are currently in the process of integrating an MHD[16,17] model into the tomography.

5.3. Thomson Scattering Remote Sensing Observations

Thomson scattering observations (photospheric sun-light scattered from electrons in the solar wind) are an alternative source of information about the solar wind. They have the advantage over *g*-level observations that they are much more directly related to the solar wind density. The line of sight integral for the Thomson scattering intensity is:

$$B = \int_0^\infty I(s) n(s) ds \tag{34}$$

where *I(s)* is the scattered intensity from a single electron. *I.e.* the intensity is a weighted mean along the line of sight of the solar wind electron density (which in turn can be easily related to the solar wind mass density assuming a neutral solar wind).

However, the heliospheric Thomson scattering signal is only a small fraction ($\leq$ 1%) of the total white light signal observed by a camera in deep space (such as the photometers on the two HELIOS spacecraft, or in Earth-orbit (such as the Solar Mass Ejection Imager[18]). In practice, it is extremely difficult to measure the total heliospheric Thomson scattering intensity. Generally, only a differential measure will be available giving the variations in Thomson scattering intensity relative to an unknown, smoothly varying Thomson scattering intensity of the 'quiet' background solar wind.

If we divide the solar wind density into two components:

$$n(s) = n_{quiet}(s) + \Delta n(s) \tag{35}$$

then the differential Thomson scattering measurements provide information only about the small variable part $\Delta n(s)$. In our tomographic algorithm the total density is needed in the solar wind model. The quiet solar wind density is specified in terms of a few parameters, *e.g.* by using a $1/r^2$ heliospheric density with an assumed, fixed value at 1 AU. With this modification Thomson scattering intensity can replace *g*-levels in the tomographic reconstruction. This has been successfully implemented using observations from the photometers on the two HELIOS spacecraft[4,5].

5.4. Time-Dependent Tomography

The algorithm presented in this paper applies to a solar wind that is essentially time-independent in the corotating reference frame. It is useful primarily to reconstruct the 'quiet' or 'background' solar wind during periods when the solar wind evolves slowly except for corotation. Usually these will be periods around the minimum of the solar cycle. Time does not occur explicitly in the formalism of this corotating tomography, and enters only implicitly through the selection of data covering a specific Carrington rotation. The basic time-resolution is one Carrington rotation, or about 27 days.

To analyze transient heliospheric events, such as coronal mass ejections, a tomography is needed that allows for the evolution of the solar wind on time scales on the order of days, *i.e.* the tomography needs to become fully time-dependent. This introduces the time dimension as an extra independent variable into the reconstruction algorithm.

The corotating tomography algorithm can loosely be described as a method to add or subtract density and velocity to the grid points at the source surface until the resulting time-independent kinematic solar wind model fits the data as well as possible. It is fairly straightforward to explicitly add the time dimension as an extra independent variable, and use the same approach, only now the source surface grid is three-dimensional, covering time as well as heliographic longitude and latitude. The heliospheric grid (Eq. 13) becomes four-dimensional with an extra time dimension:

$$t_l = T_{start} + (l-1) \times dT \qquad dT = (T_{stop} - T_{start})/(N_t - 1) \qquad (l = 1, N_t) \tag{36}$$

Similarly, the conservation laws used in the kinematic solar wind model (Section 4.2) can be extended to account for evolution over time.

Thus, the corotating algorithm can be extended to cover time-dependent solar wind conditions, such as occur during a CME event. This time-dependent tomography has been used to analyze IPS observations and HELIOS Thomson scattering observations with time steps on the order of one day[19,20]. It will play a crucial role in the analysis of Thomson scattering data from the recently launched Solar Mass Ejection Imager, which provides near-full-sky coverage over each 100-minute orbit.

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REFERENCES

1. Zidowitz, S., Inhester, B. and Epple, A., ‘The Three-dimensional Structure of the Solar Corona’, in: D. Winterhalter (ed.), *Solar Wind Eight, AIP Conf. Proc.* 382, 165–168, AIP, Woodbury, NY, 1995.
2. Frazin, R.A., ‘Tomography of the solar corona. I. A robust, regularized, positive estimation method’, *Astrophys. J.* **530**, 1026, 2000.
3. Jackson, B.V., Hick, P.L., Kojima, M., Yokobe, A., ‘Heliospheric tomography using interplanetary scintillation observations 1. Combined Nagoya and Cambridge data’, *J. Geophys. Res*. **103**, 12049, 1998.
4. Hick, P. and Jackson, B.V., ‘Three-dimensional tomography of heliospheric features using Thomson-scattering data’, *Proc. SPIE **3442***, 87, 1998.
5. Jackson, B.V. and Hick, P.P., ‘Corotational Tomography of Heliospheric Features Using Global Thomson Scattering Data’, *Solar Phys*. **211**, 345–356, 2002.
6. Hewish, A., Scott, P.F. and Wills, D., ‘Interplanetary Scintillation of Small Diameter Radio Sources’, *Nature* **203**, 1214, 1964.
7. Houminer, Z., ‘Co-rotating Plasma Streams Revealed by Interplanetary Scintillations, *Nature, Phys. Sci.* **231**, 165, 1971.
8. Hewish, A., Dennison, P.A. and Pilkington, J.D.H., ‘Measurements of the size and motion of the irregularities in the interplanetary medium’, *Nature* **209**, 1188, 1966.
9. Coles, W.A., Kaufman, J.J., ‘Solar Wind Velocity Estimation from Multi-station IPS’, *Radio Science* **13**, 591, 1978.
10. Kojima, M. and Kakinuma, T., ‘Solar Cycle Evolution of Solar Wind Speed Structure between 1973 and 1985 Observed with the Interplanetary Scintillation Method’, *J. Geophys. Res.* **92**, 7269, 1987.
11. Houminer,, Z. and Hewish, A.: ‘Correlation of Interplanetary Scintillation and Spacecraft Plasma Density Measurements’, *Planetary Space Sci.*, 1041, 1974.
12. Tappin, S.J., ‘Interplanetary scintillation and plasma density’, *Planet. Space Sci.* **34**, 93–97, 1986.
13. Gapper, G.R., Hewish, A., Purvis, A. and Duffett-Smith, P.J., ‘Observing interplanetary scintillation from the ground’, *Nature* **296**, 633, 1982.
14. Young, A.T., ‘Interpretation of interplanetary scintillation’, *Astrophys. J.* **168**, 543–562, 1971.
15. Kojima, M., Tokumaru, M., Watanabe, H., Yokobe, A., ‘Heliospheric tomography using interplanetary scintillation observations, 2. Latitude and heliocentric distance dependence of solar wind structure at 0.1-1 AU’, *J. Geophys. Res.* **103 (A2)**, 1981–1989, 1998.
16. Han, S.M., Wu, S.T., Dryer, M., ‘A Three-Dimensional, Time-Dependent, Numerical Modeling of Super-Sonic, Super-Alfvenic MHD Flow’, *Comp. & Fluids* **16**, 81, 1988.
17. Detman, T.R., Dryer, M., Yeh, T., Han, S.M., Wu, S.T., McComas, D.J., ‘Time-dependent 3-D MHD Numerical Study of Interplanetary Magnetic Draping Around Plasmoids in the Solar Wind’, *J. Geophys. Res.* **96**, 9531, 191.
18. Jackson, B.V., Buffington, A., Hick, P.L., Kahler, S.W., Simnett, G. and Webb, D.F., 1997c, ‘The Solar Mass Ejection Imager’, *Physics and Chemistry of the Earth* **22 (5)**, 441.
19. Jackson, B.V. and Hick, P. and Buffington, A., ‘Recent UCSD advances in ‘time-dependent’ tomography for use with heliospheric remote sensing data’, BAAS 31, 958, 1999.
20. Jackson, B.V. and Hick, P.P, ‘Time-dependent tomography of heliospheric features using global Thomson-scattering data from the Helios spacecraft photometers’, *BAAS* **32**, 829, 2000.

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